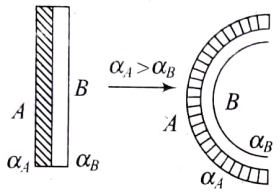
CHAPTER 1

DPP 1.1

Single Correct Answer Type

1. (c) When a copper ball is heated, its size increases. As volume $\propto$ (radius)^3 and Area $\propto$ (radius)^2, so percentage increase will be largest in its volume. Density will decrease with rise in temperature.

2. (c) A bimetallic strip on being heated bends in the form of an arc with more expandable metal (A) outside (as shown in the fiture).



3. (a) Since coefficient of expansion of steel is greater than that of bronze. Hence, with small increase in its temperature, the hole expands sufficiently.

4. (d) $A \propto L^2 \Rightarrow \dfrac{\Delta A}{A} = 2 \cdot \dfrac{\Delta L}{L} \Rightarrow \dfrac{\Delta A}{A} = 2 \times 2 = 4\%$

5. (a) The density of the liquid will decrease on heating. However, the relative change in volume of the solid will be negligible as compared to that in the density of liquid. Hence, upthrust acting on the solid will decrease and the solid will sink.

6. (d) Fractional change in period

$$\frac{\Delta T}{T} = \frac{1}{2}\alpha\Delta\theta = \frac{1}{2} \times 2 \times 10^{-6} \times 10 = 10^{-5}$$

$$\% \text{ change} = \frac{\Delta T}{T} \times 100 = 10^{-5} \times 100 = 10^{-3}\ \%$$

7. (b) $T = YA\alpha t$

Given $Y_2A_2\alpha_2t_2 - Y_1A_1\alpha_1t_1 = Y_1A_1\alpha_1t_2$

$Y_2A_2\alpha_2t_2 = Y_1A_1\alpha_1(t_1 + t_2)$

$$\frac{(t_1 + t_2)}{t_2} = \frac{Y_2A_2\alpha_2}{Y_1A_1\alpha_1}$$

or $$\frac{t_1 - t_2}{t_2} = \frac{Y_2A_2\alpha_2 - 2Y_1A_1\alpha_1}{Y_1A_1\alpha_1}$$

$$\frac{\Delta t}{t_2} = \frac{Y_2\alpha_2 - 2Y_1\alpha_1}{Y_1\alpha_1} \quad (\text{since } A_1 = A_2)$$

For $\Delta t > 0$, $Y_2\alpha_2 > 2Y_1\alpha_1$ or $\dfrac{Y_2}{2Y_1} > \dfrac{\alpha_1}{\alpha_2}$

8. (a) $\gamma_{app.} = \dfrac{\text{Mass expelled}}{\text{Mass remained} \times \Delta T}$

$$= \frac{x/100}{x \times 80} = \frac{1}{8000} = 1.25 \times 10^{-4}/^\circ\text{C}$$

9. (b) Due to volume expansion of both liquid and vessel, the change in volume of liquid relative to container is given by $\Delta V = V_0[\gamma_L - \gamma_g]\Delta\theta$

Given $V_0 = 1000$ cc, $\alpha_g = 0.1 \times 10^{-4}/^\circ\text{C}$

$\therefore \quad \gamma_g = 3\alpha_g = 3 \times 0.1 \times 10^{-4}/^\circ\text{C} = 0.3 \times 10^{-4}/^\circ\text{C}$

$\therefore \quad \Delta V = 1000[1.82 \times 10^{-4} - 0.3 \times 10^{-4}] \times 100 = 15.2$ cc

10. (a) With temperature rise (same 25°C for both), steel scale and copper wire both expand. Hence, the length of copper wire w.r.t. steel scale or the apparent length of copper wire after rise in temperature,

$$L_{app} = L'_{Cu} - L'_{steel} = [L_0(1 + \alpha_{Cu}\Delta\theta) - L_0(1 + \alpha_s\Delta\theta)$$

$$\Rightarrow \quad L_{app} = L_0(\alpha_{Cu} - \alpha_s)\Delta\theta$$

$$= 80(17 \times 10^{-6} - 11 \times 10^{-6}) \times 20 = 80.0096 \text{ cm}$$

11. (b) Loss of weight at 27°C is

$$= 46 - 30 = 16 = V_1 \times 1.24\ \rho_l \times g \qquad \text{(i)}$$

Loss of weight at 42°C is

$$= 46 - 30.5 = 15.5 = V_2 \times 1.2\ \rho_l \times g \qquad \text{(ii)}$$

Now dividing (i) by (ii), we get $\dfrac{16}{15.5} = \dfrac{V_1}{V_2} \times \dfrac{1.24}{1.2}$

But $\dfrac{V_2}{V_1} = 1 + 3\alpha(t_2 - t_1) = \dfrac{15.5 \times 1.24}{16 \times 1.2} = 1.001042$

$$\Rightarrow \quad 3\alpha(42^\circ - 27^\circ) = 0.001042 \Rightarrow \alpha = 2.316 \times 10^{-5}/^\circ\text{C}$$

12. (c) $V = V_0(1 + \gamma\Delta\theta)$

$$L^3 = L_0(1 + \alpha_1\Delta\theta)L_0^2(1 + \alpha_2\Delta\theta)^2 = L_0^3(1 + \alpha_1\Delta\theta)(1 + \alpha_2\Delta\theta)^2$$

Since $L_0^3 = V_0$ and $L^3 = V$

Hence, $1 + \gamma\Delta\theta = (1 + \alpha_1\Delta\theta)\ (1 + \alpha_2\Delta\theta)^2$

$$\cong (1 + \alpha_1\Delta\theta)(1 + 2\alpha_2\Delta\theta)$$

$$\cong (1 + \alpha_1\Delta\theta + 2\alpha_2\Delta\theta)$$

$$\Rightarrow \quad \gamma = \alpha_1 + 2\alpha_2$$

Multiple Correct Answers Type

13. (a, c)

When the disc with a central hole is heated, diameter of holes as well as outer diameter of disc both increase. As a result of this, mass of the disc will be distributed more away from its axis which means that moment of inertia will increase on heating. Now according to the law of conservation of angular momentum, as $I\omega$ = constant, so ω will decrease.

14. (a, c)

True value = Scale reading $[1 + \alpha(\theta' - \theta)]$

(a) If $\theta' > \theta$, then

Scale reading < True value

(b) $\Delta t = \dfrac{1}{2}\alpha\ \Delta\theta t$

If $\theta' > \theta$ or $\Delta\theta$ is +ve, the clock will lose time, i.e., will become slow.

(c) With rise in temperature, upthrust decreases. As weight remains same, hence a floating body sinks a little more.

(d) As upthrust acting on the body decreases with rise in temperature, hence weight of a body in a liquid $(= W_0 - Th)$ increases.

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Fill in the Blanks Type

15. $\frac{C}{5}=\frac{F-32}{9} \Rightarrow \frac{-183}{5}=\frac{F-32}{9}$

$\Rightarrow F=-297°F$

16. $\frac{F-32}{9}=\frac{K-273}{5} \Rightarrow \frac{F-32}{9}=\frac{95-273}{5} \Rightarrow F=-288°F$

17. Temperature change in Celsius scale = Temperature change in Kelvin scale = 27 K

18. Maximum density of water is at 4°C

Also $\frac{C}{5}=\frac{F-32}{9} \Rightarrow \frac{4}{5}=\frac{F-32}{9} \Rightarrow F=39.2°F$

19. $\frac{F-32}{9}=\frac{K-273}{5} \Rightarrow \frac{x-32}{9}=\frac{x-273}{5}$

$\Rightarrow x=574.25$

20. $\frac{C}{5}=\frac{F-32}{9} \Rightarrow \frac{C}{5}=\frac{(140-32)}{9} \Rightarrow C=60°$

21. $\frac{C}{5}=\frac{F-32}{9} \Rightarrow \frac{t}{5}=\frac{t-32}{9} \Rightarrow t=-40°$

22. Difference of 100°C = Difference of 180°F

$\therefore$ Difference of 30° = $\frac{180}{100}\times 30=54°$

DPP 1.2

Subjective Type

1. Heat gained by ice = Heat lost by water + Heat lost by container

Initial temperature of container = 20°C

$$4\times\frac{1}{2}\times 10+4\times 80+4\times 1x(T-0)=196\times 1\times(20-T)$$

$$20+320+4T=196\times 20-196T$$

$$200T=196\times 20-340T=\frac{3580}{200}=17.9°C$$

2. From given statement, $mgL = mL$ or $gh = L$

or $10+h=\frac{80\times 4.2}{10^{-3}}$

or $h=33.6\times 10^3$ m = 33.6 km

Single Correct Answer Type

3. (a) The volume of matter in portion AB of the curve is almost constant and pressure is decreasing. These are the characteristics of liquid state.

4. (c) Energy = $\frac{1}{2}mv^2 = mc\,\Delta\theta \Rightarrow \Delta\theta \propto v^2$

Temperature does not depend upon the mass of the balls.

5. (d) Temperature of mixture, $\theta=\frac{m_1c_1\theta_1+m_2c_2\theta_2}{m_1c_1+m_2\theta_2}$

$$\Rightarrow 32=\frac{m_1\times 0.2\times 40+100\times 0.5\times 20}{m_1\times 0.2+100\times 0.5} \Rightarrow m_1=375\text{ g}$$

6. (d) Suppose m gm ice melted, then heat required for its melting = $mL = m\times 80$ cal

Heat available with steam for being condensed and then brought to 0°C = $1\times 540+1\times 1\times(100-0)=640$ cal

$\Rightarrow$ Heat lost = Heat taken

$\Rightarrow 640 = m\times 80 \Rightarrow m = 8$ gm

7. (c) $\theta_{mix}=\frac{\frac{m_W\theta_W-m_iL_i}{c_W}}{m_i+m_W}$

$\because m_i=m_W \Rightarrow \theta_{mix}=\frac{\theta_W-\frac{L_i}{c_W}}{2}=\frac{80-\frac{336}{4.2}}{2}=0°C$

8. (a) $W=JQ \Rightarrow \frac{1}{2}\left(\frac{1}{2}Mv^2\right)=J(m.c.\Delta\theta)$

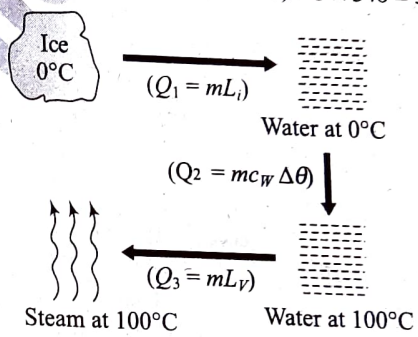
$\Rightarrow \frac{1}{4}\times 1\times(50)^2=4.2[200\times 0.105\times\times\Delta\theta] \Rightarrow \Delta\theta=7.1°C$

9. (c) Ice (0°C) converts into steam (100°C) in following three steps.

Total heat required

$Q=Q_1+Q_2+Q_3$

$=5\times 80+5+1\times(100-0)+5\times 540=3600$ cal



10. (d) Let final temperature of water be θ

Heat taken = Heat given

$110\times 1(\theta-10)+10(\theta-10)=220\times 1(170-\theta)$

$\Rightarrow \theta=48.8°C\approx 50°C$

11. (d) At 50°C, $\frac{dQ}{dt}=k(50-20)=10$

$\therefore k=\frac{1}{3}$

At an average temperature of 35°C,

$$\frac{dQ}{dt}=\frac{1}{3}(35-20)=5\text{ J/s}$$

Heat lost in 1 minute = $\frac{dQ}{dt}\times 60=300$ J

Fall in temperature = 0.2°C = $\Delta\theta$

$\therefore C=\frac{Q}{\Delta\theta}=\frac{300}{0.2°C}=1500$ J/°C

12. (a) Let V cc be the volume of either liquid.

Thermal capacity of each calorimeter is V cap per °C.

Mass of water = $V\times 1 = V$ g

Mass of alcohol = $V\times 0.8 = 0.8\,V$ g

Rate of cooling of water,

$=\frac{1}{100}[V(50-40)+V\times 1\times(50-40)]=\frac{V}{5}$ cal/sec

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Rate of cooling of alcohol

$$= \frac{1}{74}[V(50-40) + S \times 0.8V \times 10]$$

$$= \frac{1}{74}[10V + 8V \times S]$$

Now $\frac{1}{74}[10V + 8V \times S] = \frac{V}{5}$

$\therefore \quad s = 0.6$ cal/g-°C

13. (a) As the system is thermally isolated, the heat required for vaporization is extracted from water itself. The water in the vessel being at 0°C results in part of water getting frozen. Let x be the maximum fraction of water in this way. Then $(1 - x)$ is the fraction that gets vaporized.

$\therefore \quad x \times 3.36 \times 10^5 = (1 - x) \times 21 \times 10^5$

or $\quad x = 0.862$

Multiple Correct Answers Type

14. (a, b, c)

Q_1 = Heat given out by water if it was to cool up to 0°C
$= mg \times 1$ cal/g-°C $\times (10 - 0) = 10$ mcal

Q_2 = Heat required by ice to melt completely into water at 0°C
$= m \times L_F = mg \times 80$ cal/g $= 80$ mcal

Fill in the Blanks Type

15. Heat taken by ice to melt at 0°C is

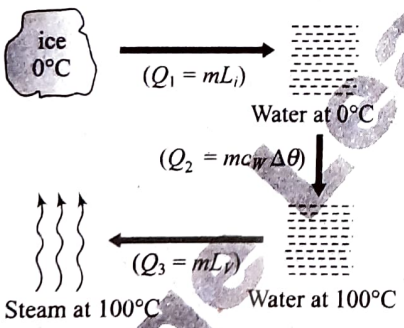
$Q_1 = mL = 540 \times 80 = 43200$ cal

Heat given by water to cool upto 0°C is

$Q_2 = ms\Delta\theta = 540 \times 1 \times (80 - 0) = 43200$ cal

Hence heat given by water is just sufficient to melt the whole ice and final temperature of mixture is 0°C.

16. Conversion of ice (0°C) into steam (100°C) is as follows:



Heat required in the given process $= Q_1 + Q_2 + Q_3$
$= 1 \times 80 + 1 \times 1 \times (100 - 0) + 1 \times 536 = 716$ cal

17. If m gm ice melts, then

Heat lost = Heat gain

$80 \times 1 \times (30 - 0) = m \times 80 \Rightarrow m = 30$ gm

18. Thermal capacity = Mass × Specific heat

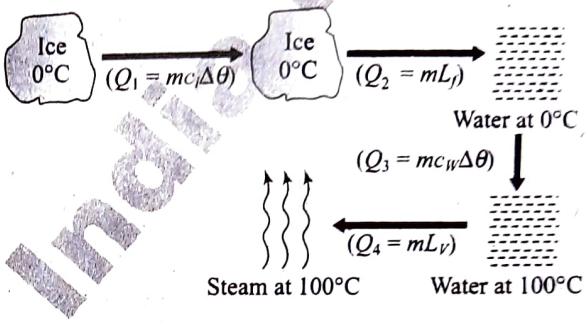
Due to same material, both spheres will have same specific heat. Also mass = Volume (V) × Density (ρ)

$\therefore$ Ratio of thermal capacity

$$= \frac{m_1}{m_2} = \frac{V_1\rho}{V_2\rho} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1}{2}\right)^3 = 1:8$$

19. Ice (–10°C) converts into steam as follows

(c_i = Specific heat of ice, c_W = Specific heat of water)



Total heat required $Q = Q_1 + Q_2 + Q_3 + Q_4$

$\Rightarrow \quad Q = 1 \times 0.5(10) + 1 \times 80 + 1 \times 1 \times (100 - 0) + 1 \times 540$
$= 725$ cal

Hence, work done, $W = JQ = 4.2 \times 725 = 3045$ J

20. $W = JQ \Rightarrow mgh = J \times Q$

$$\Rightarrow \quad Q = \frac{mgh}{J} = \frac{5 \times 9.8 \times 30}{4.2} = 350 \text{ cal}$$

21. $W = JQ = 4.18 \times 400 = 1672$ J

22. Energy supplied $= 0.93 \times 3600$ J $= 3348$ J

Heat required to melt 10 gms of ice
$= 10 \times 80 \times 4.18 = 3344$ J

Hence block of ice just melts.

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CHAPTER 2

DPP 2.1

Single Correct Answer Type

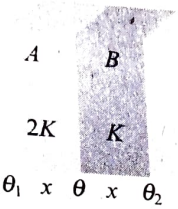
1. (c) Temperature of interface

$$\theta = \frac{K_1\phi_1 l_2 + K_2\phi_2 l_1}{K_1 l_2 + K_2 l_1} = \frac{K\times 0\times 2 + 3K\times 100\times 1}{K\times 2 + 3K\times 1} = 60°C$$

2. (b) Suppose thickness of each wall is x. Then

$$\left(\frac{Q}{t}\right)_{\text{combination}} = \left(\frac{Q}{t}\right)_A$$

$$\Rightarrow \quad \frac{K_S A(\theta_1-\theta_2)}{2x} = \frac{2KA(\theta_1-\theta)}{x}$$



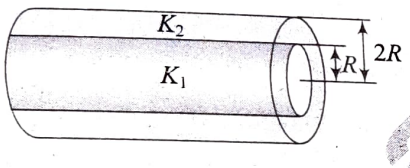
$$\because \quad K_S = \frac{2\times 2K\times K}{(2K+K)} = \frac{4}{3}K$$

and $(\theta_1-\theta_2) = 36°$

$$\Rightarrow \quad \frac{\frac{4}{3}KA\times 36}{2x} = \frac{2KA(\theta_1-\theta)}{x}$$

Hence temperature difference across wall A is $(\theta_1-\theta_2) = 12°C$

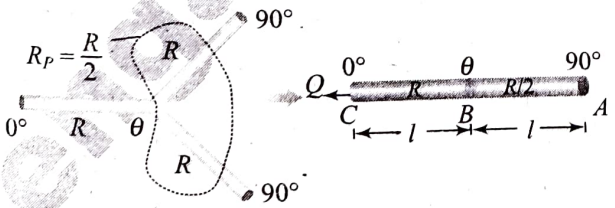
3. (c) Both the cylinders are in parallel, for the heat flow from one end as shown.



Hence $K_{eq} = \frac{K_1A_1 + K_2A_2}{A_1 + A_2}$; where A_1 = area of cross-section of inner cylinder = πR^2 and A_2 = area of cross-section of cylindrical shell = $\pi(2R)^2 - (R)^2 = 3\pi R^2$

$$\Rightarrow \quad K_{eq} = \frac{K_1(\pi R^2) + K_2(3\pi R^2)}{\pi R^2 + 3\pi R^2} = \frac{K_1 + 3K_2}{4}$$

4. (b) Let the temperature of junction be θ. Since roads B and C are parallel to each other (because both having the same temperature difference), hence given figure can be redrawn as follows:



$$\because \quad \frac{Q}{t} = \frac{(\theta_1-\theta_2)}{R} \text{ and } \left(\frac{Q}{t}\right)_{AB} = \left(\frac{Q}{t}\right)_{BC}$$

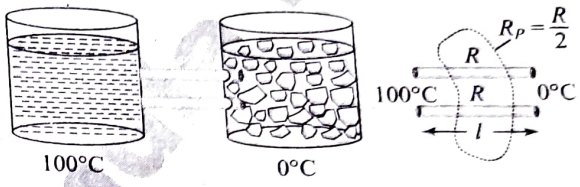
$$\Rightarrow \quad \frac{(90-\theta)}{R/2} = \frac{(\theta-0)}{R} \Rightarrow 180 - 2\theta = \theta \Rightarrow \theta = 60°C$$

5. (a) The top surface of the ice is at $T_C = -5.0°C$, and the bottom of the body of water is at $T_H = 4.0°C$ and the interface between the ice and the water is at $T_X = 0.0°C$. The rate of heat flow through both layers should be equal

$$\frac{k_{\text{water}}A(T_H - T_X)}{L - L_{\text{ice}}} = \frac{k_{\text{ice}}A(T_X - T_C)}{L_{\text{ice}}}$$

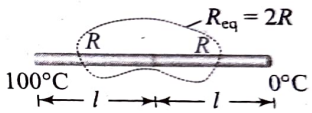
We cancel the area A and solve for thickness of the ice layer: $L_{\text{ice}} = 1.1$ m.

6. (c) Initially the rods are placed in vessels as shown below



$$\frac{Q}{t} = \frac{(\theta_1-\theta_2)}{R} \Rightarrow \left(\frac{Q}{t}\right)_1 = \frac{mL}{t} = q_1L = \frac{(100-0)}{\frac{R}{2}} \qquad \text{(i)}$$

Finally when rods are joined end to end as shown



$$\Rightarrow \quad \left(\frac{Q}{t}\right)_2 = \frac{mL}{t} = q_2L = \frac{(100-0)}{2R} \qquad \text{(ii)}$$

From equation (i) and (ii), $\frac{q_1}{q_2} = \frac{4}{1}$

7. (b) Rate of cooling of a body $R = \frac{\Delta\theta}{t} = \frac{A\varepsilon\sigma(T^4 - T_0^4)}{mc}$

$$\Rightarrow \quad R \propto \frac{A}{m} \propto \frac{\text{Area}}{\text{Volume}}$$

$\Rightarrow$ For the same surface area, $R \propto \frac{1}{\text{Volume}}$

$\because$ Volume of cube < Volume of sphere

$\Rightarrow$ $R_{\text{Cube}} > R_{\text{Sphere}}$, i.e., cube cools down with faster rate

8. (b) $Q = mL = KA\frac{(\theta_1-\theta_2)}{l}t$

$$\Rightarrow \quad m = \frac{1}{L}\times KA\frac{(\theta_1-\theta_2)}{l}\times t$$

$$= \frac{1}{80}\times 0.2\times 4\times\frac{(100-0)}{\sqrt{4}}\times 10\times 60$$

$(\because l^2 = 4 \Rightarrow l =$

$$= \frac{0.2\times 4\times 100\times 600}{80\times 2} = 300 \text{ gm}$$

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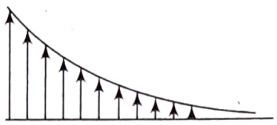
9. (b) $H = -kA\dfrac{dT}{dx} \Rightarrow \dfrac{dT}{dx} = \dfrac{-H}{kA}$

Now as k increases, $\dfrac{dT}{dx}$ becomes less (–)ve.

So slope becomes less (–)ve. So the curve will be



Multiple Correct Answers Type

10. (b, d)

K_{eq} is same in all three cases. All other parameters being same, rate of energy conduction is same in all three cases.

Similarly temperature difference across any material in any wall is also same.

11. (a, c)

Heat obviously flows from higher temperature to lower temperature in steady state. ⇒ (a) is true.

Temperature gradient $\propto \dfrac{1}{\text{Cross-section area}}$ in steady state. ⇒ (b) is false.

Thermal current through each cross section area is same.

⇒ (c) is true.

Temperature decreases along the length of the rod from higher temperature end to lower temperature end. ⇒ (d) is false.

Comprehension Type

12. (d) We find the rate of energy conducted upward to be

$$P_{cond} = \frac{Q}{t} = kA\frac{T_H - T_C}{L} = (0.400\ \text{W/m}\cdot\text{°C})A\frac{5.0\text{°C}}{0.10\text{m}}$$

$$= (20.0 \times 0.5)\,\text{W} = 10\,\text{W}$$

Recall that a change in Celsius temperature is numerically equivalent to a change on the Kelvin scale.

13. (b) The heat of fusion in this process is $Q = mL_F$ where $L_F = 4.0 \times 10^5$ J/K. Differentiating the expression with respect to t and equating the result with P_{cond}, we have $P_{cond} = \dfrac{dQ}{dt} = L_F\dfrac{dm}{dt}$.

Thus, the rate of mass converted from liquid to ice is

$$\frac{dm}{dt} = \frac{P_{cond}}{L_F} = \frac{10.0\,\text{W}}{4.0\times10^5\,\text{J/kg}} = 2.5\times10^{-5}\,\text{kg/s}$$

14. (c) Since $m = \rho V = \rho Ah$, differentiating both sides of the expression gives

$$\frac{dm}{dt} = \frac{d}{dt}(\rho Ah) = \rho A\frac{dh}{dt}$$

Thus, the rate of change of the icicle length is

$$\frac{dh}{dt} = \frac{1}{\rho A}\frac{dm}{dt} = \frac{2.5\times10^{-5}\,\text{kg/m}^2\cdot\text{s}}{1000\,\text{kg/m}^3} = 2.5\times10^{-8}\,\text{m/s}$$

Fill in the Blanks Type

15. $\dfrac{\Delta Q}{\Delta t} = \dfrac{KA\Delta\theta}{\Delta x} \Rightarrow$ Thermal gradient $= \dfrac{\Delta\theta}{\Delta x}$

$$= \frac{(\Delta Q/\Delta t)}{KA} = \frac{10}{0.4} = 25\text{°C/cm}$$

16. $Q = \dfrac{KA(\theta_1 - \theta_2)t}{l}$; in both the cases , A, l and $(\theta_1 - \theta_2)$ are same

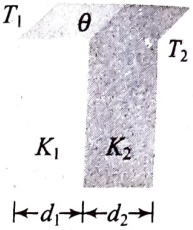
so Kt = constant $\Rightarrow \dfrac{K_1}{K_2} = \dfrac{t_1}{t_2} = \dfrac{30}{20} = \dfrac{3}{2} = 1.5$.

17. In series both walls have same rate of heat flow. Therefore

$$\frac{dQ}{dt} = \frac{K_1A(T_1 - \theta)}{d_1} = \frac{K_2A(\theta - T_2)}{d_2}$$

$$\Rightarrow K_1d_2(T_1 - \theta) = K_2d_1(\theta - T_2)$$

$$\Rightarrow \theta = \frac{K_1d_2T_1 + K_2d_1T_2}{K_1d_2 + K_2d_1}$$



18. Thermal conductivity of composite plate

$$K_{eq} = \frac{2K_1K_2}{K_1 + K_2} = \frac{2\times2\times3}{2+3} = \frac{12}{5} = 2.4$$

19. $\dfrac{Q}{t} = \dfrac{KA(\Delta\theta)}{l} \Rightarrow \dfrac{Q}{t} \propto \dfrac{A}{l} \propto \dfrac{r^2}{l}$.

$$\Rightarrow \frac{(Q/t)_1}{(Q/t)_2} = \left(\frac{r_1}{r_2}\right)^2 \times \frac{l_2}{l_1} = \left(\frac{2}{1}\right)^2 \times \left(\frac{4}{1}\right) = \frac{16}{1}$$

20. Temperature of interface, $\theta = \dfrac{K_1\theta_1 + K_2\theta_2}{K_1 + K_2}$

Where $K_1 = 2K$ and $K_2 = 3K$ $\left(\because \dfrac{K_1}{K_2} = \dfrac{2}{3}\right)$

$$\Rightarrow \theta = \frac{2K\times100 + 3K\times0}{2K + 3K} = \frac{200K}{5K} = 40\text{°C}$$

DPP 2.2

Single Correct Answer Type

1. (b) Black cloth is a good absorber of heat, therefore ice covered by black cloth melts more as compared to that covered by white cloth.

2. (a) As for a black body, rate of absorption of heat is more. Hence thermometer A shows faster rise in temperature but finally both will acquire the atmospheric temperature.

3. (b) Absorption power $= \dfrac{\text{Heat absorbed}}{\text{Total heat given}}$

4. (d) The total energy radiated from a black body per minute.

$$Q \propto T^4 \Rightarrow \frac{Q_2}{Q_1} = \left(\frac{2T}{T}\right)^4 = 16 \Rightarrow Q_2 = 16Q_1$$

If m be the mass of water taken and S be its specific heat capacity, then $Q_1 = ms(20.5 - 20)$ and $Q_2 = m_s(\theta - 20)$

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θ°C = Final temperature of water

$$\Rightarrow \frac{Q_2}{Q_1}=\frac{\theta-20}{0.5} \Rightarrow \frac{16}{1}=\frac{\theta-20}{0.5} \Rightarrow \theta=28°C$$

5. (a) r = Radius of planet

T_1 = Temperature of planet at which energy is radiated

Power radiated by Sun, $P = 4\pi R^2\sigma T_0^4$

Energy received by planet = $\frac{P}{4\pi d^2}\times \pi r^2 = \frac{R^2T_0^4\pi r^2\sigma}{d^2}$

Energy radiated by planet = $4\pi r^2\sigma T_1^4$

For thermal equilibrium, $\frac{R^2T_0^4\pi r^2\sigma}{d^2} = 4\pi r^2\sigma T_1^4$

$$R^2T_0^4 = 4d^2T_1^4$$

$$T_1 = T_0\sqrt{\frac{R}{2d}}$$

6. (c) $T_1 = T_0\sqrt{\frac{2R}{2\times 2d}} = \frac{T_0}{2}\sqrt{\frac{2R}{d}} = \frac{T_0}{2}\theta^{1/2}$

7. (d) Let T_0 = initial temperature of the black body

$\therefore \quad \lambda_0T_0 = b$ (constant)

Power radiated = $P_0 = CT_0^4$ (C is a constant)

Let T = new temperature of the black body

$$\therefore \quad \frac{3\lambda_0}{r}T = b = \lambda_0T_0 \quad \text{or} \quad T = \frac{4T_0}{3}$$

Power radiated, $CT^4 = CT_0^4\left(\frac{4}{3}\right)^4 = \left(\frac{256}{81}\right)P_0$

Multiple Correct Answers Type

8. (a, c)

According to Wien's law,

$$\lambda_mT = \text{constant} \quad \text{or} \quad \frac{T}{\nu_m} = \text{constant}$$

So if T is doubled, ν_m is also doubled.

Further, according to Stefan's law, $E \propto T^4$

When T is doubled, E increases by a factor of 16.

9. (a, c)

R = radius of Sun,

d = distance of the Earth from the Sun

Power radiated from the Sun = $(4\pi R^2)\sigma T^4 = P$

Energy received/area/sec, normally on the earth

$$= S = \frac{P}{4\pi d^2} = \frac{4\pi R^2\sigma T^4}{4\pi d^2} = \sigma T^4\frac{R^2}{d^2} = \frac{1}{4}\sigma T^4\left(\frac{2R}{d}\right)^2$$

Angle subtended by the Sun at the Earth = $\alpha = \frac{2R}{d}$

or $S = \text{Constant}\times T^4\times\alpha^2$

Comprehension Type

10. (b) Eventually the body will attain temperature T_2.

11. (b) The rate of emission depends upon emissivity.

12. (c) The body must emit energy at the same rate at which it absorbs it.

13. (b) Striking energy is either absorbed or reflected. Thus, a good absorber is a poor reflector.

Fill in the Blanks Type

14. $\lambda_mT = \text{Constant} \Rightarrow \frac{T_1}{T_2} = \frac{\lambda_2}{\lambda_1} \Rightarrow \frac{10^{-4}}{0.5\times 10^{-5}} = 200$

15. $\lambda_{m_2} = \frac{T_1}{T_2}\times\lambda_{m_1} = \frac{1500}{2500}\times 5000 = 3000$ Å

16. $\frac{T_S}{T_N} = \frac{(\lambda_N)_{max}}{(\lambda_S)_{max}} = \frac{350}{510} = 0.69$

17. $E\propto T^4 \Rightarrow \frac{E_1}{E_2} = \frac{T^4}{T^4\times 2^4} \Rightarrow E_2 = \frac{E}{16}$

18. $\frac{E_2}{E_1} = \left(\frac{T_2}{T_1}\right)^4$

$$\Rightarrow T_2 = \left(\frac{E_2}{E_1}\right)^{1/4}\times T_1 = (16)^{1/4}\times(273+127)$$

$$\Rightarrow T_2 = 800\text{ K} = 527°C$$

19. $(Q)_{\text{Black body}} = A\sigma T^4t \Rightarrow \frac{Q}{t}\propto P = A\sigma T^4$

Breadths are halved, so area becomes one-fourth.

$$\Rightarrow \frac{P_1}{P_2} = \frac{A_1}{A_2}\times\left(\frac{T_1}{T_2}\right)^4 \Rightarrow \frac{A_1}{(A_1/4)}\times\left(\frac{273+327}{273+127}\right)$$

$$\Rightarrow P_2 = \frac{81}{64}E$$

DPP 2.3

Single Correct Answer Type

1. (a) Initially at $t = 0$

Rate of cooling (R) $\propto$ Fall in temperature of body $(\theta-\theta_0)$

$$\Rightarrow \frac{R_1}{R_2} = \frac{\theta_1-\theta_0}{\theta_2-\theta_0} = \frac{100-40}{80-40} = \frac{3}{2}$$

2. (b) Liquid having more specific heat has slow rate of cooling because for equal masses rate of cooling $\Rightarrow \frac{d\theta}{dt}\propto\frac{1}{c}$.

3. (b) $\frac{\theta_1-\theta_2}{t} = K\left(\frac{\theta_1+\theta_2}{2}-\theta_0\right)$

$$\therefore \quad \frac{100-70}{4} = K\left(\frac{100+70}{2}-15\right) = 60K \Rightarrow K = \frac{1}{8}$$

Again $\frac{70-40}{t} = \frac{1}{8}\left(\frac{70+40}{2}-15\right) = 5 \Rightarrow t = 6$ min

4. (c) $\frac{60-50}{10} = K\left(\frac{60+50}{2}-25\right)$ (i)

$$\frac{50-\theta}{10} = K\left[\frac{50+\theta}{2}-25\right] \quad \text{(ii)}$$

On dividing, we get $\frac{10}{50-\theta} = \frac{60}{\theta} \Rightarrow \theta = 42.84°C$

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5. (a) $\frac{365-361}{2}=K\left[\frac{365+361}{2}-293\right]=70K \Rightarrow K=\frac{1}{35}$

Again $\frac{344-342}{t}=\frac{1}{35}\left[\frac{344-342}{2}-293\right]=\frac{10}{7}$

$\Rightarrow \quad t=\frac{14}{10}\text{min}=\frac{14}{10}\times 60=84\text{ sec}$

6. (b) Rate of cooling $=\frac{-d\theta}{dt}\propto\left(\frac{\theta_1+\theta_2}{2}-\theta_0\right)$

In second case average temperature will be less hence rate of cooling will be less. Therefore time taken will be more than 4 minutes.

7. (b) First case, $\frac{61-59}{4}=K\left[\frac{61+59}{2}-30\right]$ (i)

Second case, $\frac{51-49}{t}=K\left[\frac{51+49}{2}-30\right]$ (ii)

By solving equation (i) and (ii) we get t = 6 min.

8. (d) Rate of cooling (here it is rate of loss of heat)

$$\frac{dQ}{dt}=(mc+W)\frac{d\theta}{dt}=(m_lc_l+m_cc_c)\frac{d\theta}{dt}$$

$$\Rightarrow \quad \frac{dQ}{dt}=(0.5\times 2400+0.2\times 900)\left(\frac{60-55}{60}\right)=115\frac{\text{J}}{\text{sec}}.$$

9. (b) According to Newton's law of cooling.

Comprehension Type

10. (b) Temperature of Sun's core is $T=\frac{b}{\lambda_m}$

Rate of energy radiated by Sun

$$=\sigma(4\pi R^2)\left(\frac{b}{\lambda_m}\right)^4=\frac{\Delta m}{t}c^2$$

$$\frac{\left(\begin{array}{l}\text{net energy obtained from conversion of}\\ \text{mass of Hydrogen in Sun's core}\end{array}\right)}{\text{total time.}}=\frac{(\eta m)c^2}{t}$$

$$\Rightarrow \quad t=\frac{\eta mc^2\lambda_m^4}{\sigma(4\pi R^2)b^4}$$

11. (a) Power radiated by sun $=P=(\sigma)(4\pi R^2)\left(\frac{b}{\lambda_m}\right)^4$

Intensity falling at earth $=\frac{(\sigma)(4\pi R^2)(b/\lambda_m)^4}{4\pi d^2}$

$$S=\sigma\left(\frac{R^2}{d^2}\right)\left(\frac{b}{\lambda_m}\right)^4$$

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CHAPTER 3

DPP 3.1

Single Correct Answer Type

1. (d) Ideal gas equation $PV = \mu RT = \left(\frac{N}{N_A}\right)RT$ where N = Number of molecule, N_A = Avogadro number

$$\therefore \quad \frac{N_1}{N_2} = \left(\frac{P_1}{P_2}\right)\left(\frac{V_1}{V_2}\right)\left(\frac{T_2}{T_1}\right) = \left(\frac{P}{2P}\right)\left(\frac{V}{V/4}\right)\left(\frac{2T}{T}\right) = \frac{4}{1}$$

2. (a) For oxygen $v_{O_2} = \sqrt{\frac{3RT_{O_2}}{M_{O_2}}}$ and

For hydrogen $v_{H_2} = \sqrt{3R\frac{T_{H_2}}{M_{H_2}}}$

According to problem $= \sqrt{\frac{3RT_{O_2}}{M_{O_2}}} = \sqrt{3R\frac{T_{H_2}}{M_{H_2}}}$

$$\Rightarrow \quad \frac{T_{O_2}}{M_{O_2}} = \frac{T_{H_2}}{M_{H_2}} \quad \Rightarrow \quad \frac{47+273}{32} = \frac{T_{H_2}}{2}$$

$$\Rightarrow \quad T_{H_2} = \frac{320}{32} \times 2 = 20 \text{ K}$$

3. (b) $P = \frac{1}{3}\frac{mN}{V}v_{rms}^2 \quad \therefore \quad P \propto mv_{rms}^2$

So $\frac{P_2}{P_1} = \frac{m_2}{m_1} \times \left(\frac{v_2}{v_1}\right)^2 = \frac{m_1/2}{m_1}\left(\frac{2v_1}{v_1}\right)^2 = 2$

$$\Rightarrow \quad P_2 = 2P_1 = 2P_0$$

4. (c) $v_{rms} = \sqrt{\frac{3RT}{M}}$. According to problem T will becomes $T/2$ and M will become $M/2$ so the value of v_{rms} will increase by $\sqrt{4} = 2$ times i.e. new root mean square velocity will be $2v$.

5. (c) Change in pressure will not affect the rms velocity of molecules. So we will calculate only the effect of temperature.

As $v_{rms} \propto \sqrt{T}$

$$\therefore \quad \frac{v_{300^\circ}}{v_{400^\circ}} = \sqrt{\frac{300}{400}} = \sqrt{\frac{3}{4}} \quad \Rightarrow \quad \frac{200}{v_{400}} = \sqrt{\frac{3}{4}}$$

$$\Rightarrow \quad v_{400} = \frac{200 \times 2}{\sqrt{3}} = \frac{400}{\sqrt{3}} \text{ m/s}$$

6. (a) At absolute temperature, kinetic energy of gas molecules becomes zero but they possess potential energy so we can say that absolute zero degree temperature is not zero energy temperature.

7. (c) $E \propto T \quad \therefore \quad \frac{E_2}{E_1} = \frac{T_2}{T_1} \quad \Rightarrow \quad \frac{2E_1}{E_1} = \frac{T_2}{(20+273)}$

$$\Rightarrow \quad T_2 = 293 \times 2 = 586 \text{ K} = 313^\circ\text{C}$$

8. (a) Kinetic energy per degree of freedom $= \frac{1}{2}kT$

As diatomic gas possess two degree of freedom for rotational motion therefore rotational K.E. $= 2\left(\frac{1}{2}kT\right) = kT$

In the problem both gases (oxygen and nitrogen) are diatomic and have same temperature (300 K) therefore ratio of average rotational kinetic energy will be equal to one.

9. (b) Average speed of gas molecule, $v_{av} = \sqrt{\frac{8kT}{\pi m}}$. It depends on temperature and molecular mass. So the average speed of oxygen will be same in vessel A and vessel C and that is equal to V_1.

Comprehension Based

10. (a) Pressure of the gas

$$= \frac{1}{3}\cdot\frac{mN}{AL}\cdot 3U_x^2 = \frac{m\langle U_x^2\rangle N}{AL}$$

11. (c) Momentum transferred per collision x no. of collisions $= F$

$$P = \frac{F}{A} = \frac{\Delta p \times f}{A}$$

12. (c) Both increases as V_{rms} increases.

13. (b) As temp. is not changed, V_{rms} does not change. Hence, frequency of collisions is doubled due to the length of round trip which is halved.

14. (d) Pressure change is due to area of the walls becoming half.

Fill in the Blanks Type

15. As $P = \frac{1}{3}\rho v_{rms}^2 = \frac{1}{3}(8.99 \times 10^{-2}) \times (3180)^2$

$$= 3.03 \times \text{N/m}^2 = 3.0 \text{ atm}$$

16. Kinetic energy $E = 1.5 \times 10^5$ J,
volume $V = 20$ litre $= 20 \times 10^{-3}$ m^3

$$\text{Pressure} = \frac{2}{3}\frac{E}{V} = \frac{2}{3}\left(\frac{1.5 \times 10^5}{20 \times 10^{-3}}\right) = 5 \times 10^6 \text{ N/m}^2$$

17. $V = 10^{-3}$ m^3, $N = 3.0 \times 10^{22}$
$m = 5.3 \times 10^{-26}$ kg, $v_{rms} = 400$ m/s

$$P = \frac{1}{3}\frac{mN}{V}v_{rms}^2 = \frac{1}{3} \times \frac{5.3 \times 10^{-26} \times 3.0 \times 10^{22}}{10^{-3}}(400)^2$$

$$= 8.48 \times 10^4 \text{ N/m}^2.$$

18. From $PV = \mu RT$

we get $\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)\left(\frac{V_1}{V_2}\right) = \left(\frac{T_1/2}{T_1}\right)\left(\frac{V_1}{2V_1}\right) = \frac{1}{4}$

$$\Rightarrow \quad P_2 = \frac{P_1}{4}$$

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19. As $v_{rms} = \sqrt{\frac{3RT}{M}}$

$$\therefore \quad \frac{C_A}{C_B} = \sqrt{\frac{T_A/T_B}{M_A/M_B}} = \sqrt{4} = 2 \qquad \left[\text{As } \frac{T_A}{T_B} = 4\frac{M_A}{M_B} \text{ given}\right]$$

20. Root mean square velocity does not depends upon the quantity of gas. For a given gas and at constant temperature, it always remains same.

21. $v_{rms} = \sqrt{\frac{3RT}{M}}$

$$\therefore \quad \frac{v_{H_2}}{v_{O_2}} = \sqrt{\frac{T_{H_2}}{M_{H_2}} \times \frac{M_{O_2}}{T_{O_2}}} \Rightarrow \frac{1930}{v_{O_2}} = \sqrt{\frac{300}{2} \times \frac{32}{900}}$$

$$\Rightarrow \quad v_{O_2} = \frac{1930 \times \sqrt{3}}{4} = 836 \text{ m/s}$$

22. $v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + v_4^2 + v_5^2}{5}}$

$$= \sqrt{\frac{2^2 + 3^2 + 4^2 + 5^2 + 6^2}{5}} = \sqrt{\frac{100}{5}} = \sqrt{20} = 4.24$$

DPP 3.2

Single Correct Answer Type

1. (b) $VP^3 = \text{constant} = k \Rightarrow P = \frac{k}{V^{1/3}}$

Also $PV = \mu RT \Rightarrow \frac{k}{V^{1/3}} \cdot V = \mu RT \Rightarrow V^{2/3} = \frac{\mu RT}{k}$

Hence $\left(\frac{V_1}{V_2}\right)^{2/3} = \frac{T_1}{T_2} \Rightarrow \left(\frac{V}{27V}\right)^{2/3} = \frac{T}{T_2} \Rightarrow T_2 = 9T$

2. (c) $\mu = \mu_1 + \mu_2$

$$\frac{P(2V)}{RT_1} = \frac{P'v}{RT_1} + \frac{P'V}{RT_2} \Rightarrow \frac{2P}{RT_1} = \frac{P'}{R}\left[\frac{T_2 + T_1}{T_1 T_2}\right]$$

$$P' = \frac{2PT_2}{(T_1 + T_2)} = \frac{2 \times 1 \times 600}{(300 + 600)} = \frac{4}{3} \text{ atm}$$

3. (c) $\rho = \frac{PM}{RT}$

Density ρ remains constant when P/T or volume remains constant.

In graph (i), pressure is increasing at constant temperature hence volume is decreasing so density is increasing. Graphs (ii) and (iii) volume is increasing, hence, density is decreasing. Note that volume would have been constant in case the straight line in graph (iii) had passed through origin.

4. (c) $\rho \propto \frac{1}{V}$ in the process AB.

(V decreases, ρ increases).

In isothermal process (T = constant), $\rho \propto \frac{1}{V}$.

As V decreases, P increases.

So, for ρ increase, V decreases (ρV curve) and P increases for V decreases ($P - T$ curve)

Options (b) and (c) may be correct.

But process B – C is shown by a dot in (ρ-V) curve.

It means BC is a isochoric process.

Process BC in option (c) is a isochoric process because $P \propto T$.

So, option (c) is correct.

5. (d) Volume of gas is constant

$$T_1/T_2 = P_1/P_2$$

$$\frac{100}{(100 + \Delta T)} = \frac{P}{1.05P}$$

$$105P = 100P + P.\Delta T$$

$$\frac{5P}{P} = \Delta T = 5°C$$

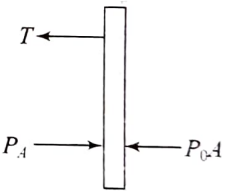
6. (d) Using Boyle's law, we have $\frac{V}{T} = \text{constant}$

$$\Rightarrow \quad \frac{\frac{l}{2} + 5}{373} = \frac{\frac{l}{2} - 5}{273}$$

As the piston moves 5 cm, the length of one side will be $\left(\frac{l}{2} + 5\right)$ and other side $\left(\frac{l}{2} - 5\right)$. On solving this equation, we get $l = 64.6$ cm.

7. (c) $T + P_0A = PA$

$T = (P - P_0)A = \frac{3}{8}P_0A$ (given)



$$P = \frac{3}{8}P_0 + P_0 = \frac{11}{8}P_0$$

Now volume is constant by string.

So, $P \propto T$

Initial temperature is T_0.

Now as pressure has become $\frac{11}{8}P_0$, temperature will be $\frac{11}{8}T_0$.

8. (c) $PV = \frac{m}{M}RT$

$$V = \left(\frac{mR}{M}\right)\left(\frac{T}{P}\right) \text{ or } V \propto \left(\frac{T}{P}\right)$$

$$\left(\frac{T}{P}\right)_A = \frac{T_0}{2P_0} \text{ and } \left(\frac{T}{P}\right)_B = \frac{T_0}{2P_0}$$

$$\left(\frac{T}{P}\right)_C = \frac{3T_0}{2P_0} \text{ and } \left(\frac{T}{P}\right)_D = T_0/P_0$$

Volume decreases form C to D.

9. (b) Since hydrogen diffuses through all the partitions, it uniformly spreads over the entire vessel and in all the three parts of the vessel. The pressure of hydrogen

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$$P_{H_2} = \frac{m_{H_2}}{M_{H_2}} \frac{RT}{V}$$

Nitrogen can diffuse only through and hence will fill the middle and right parts having volume $2V/3$. The pressure of N_2 is

$$P_{N_2} = \frac{m_{N_2}}{M_{N_2}} = \frac{3RT}{V}$$

Oxygen does not diffuse through partitions and its pressure in the middle part of vessel

$$P_{O_2} = \frac{m_{O_2}}{M_{O_2}} \frac{3RT}{V}$$

According to Dalton's law, pressure in a part of the vessel is equal to the sum of partial pressures of gases it contains.

$$P_1 = P^{H_2} = 1.3 \times 10^9 \text{ Pa}$$

$$P_2 = P_{H_2} + P_{O_2} + P_{N_2} = 4.5 \times 10^9 \text{ Pa}$$

$$P_3 = P_{H_2} + P_{N_2} = 2 \times 10^9 \text{ Pa}$$

$$\therefore \quad P_1 : P_2 : P_3 = 1.3 : 4.5 : 2$$

Multiple Correct Answers Type

10. (a, b, c)

We use $pV = nRT$. The volume of the tank is

$$V = \frac{nRT}{p} = \frac{\left(\frac{300\,\text{g}}{17\,\text{g/mol}}\right)(8.31\,\text{J/mol}\cdot\text{K})(350\,\text{K})}{1.35 \times 10^6\,\text{Pa}}$$

$$= 3.8 \times 10^{-2}\,\text{m}^3 = 38\,\text{L}$$

The number of moles of the remaining gas is

$$n' = \frac{p'V}{RT'} = \frac{(8.7 \times 10^5\,\text{Pa})(3.8 \times 10^{-2}\,\text{m}^3)}{(8.31\,\text{J/mol}\cdot\text{K})(293\,\text{K})} = 13.5\,\text{mol}$$

The mass of the gas that leaked out is then

$\Delta m = 300\,\text{g} - (13.5\,\text{mol})(17\,\text{g/mol}) = 71\,\text{g}$

11. (a, c)

In 1 mole of all the gases, there are N (Avagdro's number) number of molecules which are independent of the nature of the gas. Hence, option (a) is correct. The transitional kinetic energy of 1 mole of any gas at temperature T is $\frac{3}{2}RT$. Therefore, option (c) is also correct.

12. (c, d)

Internal energy of n moles of an ideal gas is

$$U = \left(\frac{f}{2}\right)(nRT)$$

i.e., U depends on the number of moles n and temperature T.

Similarly, $PV = nRT$, i.e., PV depends on n and T.

But $\frac{P}{\rho} = \frac{RT}{M}$, i.e., the ratio P/ρ depends on T only.

Similarly, $V_{rms} = \sqrt{\frac{3RT}{M}}$, i.e., v_{rms} also depends on T only.

Comprehension Type

13. (c), 14. (b)

From the P–V graph, the relation between P and V is

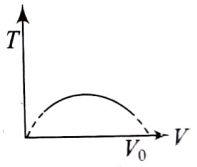
$$P = -\frac{P_0}{V_0}V + P_0 \qquad (1)$$

Also the ideal gas state equation for one mole is $PV = RT$ (2)

From equation (1) and (2) is $T = \frac{P_0}{R}V\left(1 - \frac{V}{V_0}\right)$

Hence the graph of T vs. V is a parabola given by

$$P - V \quad P = -\frac{P_0}{V_0}V + P_0$$



Obviously T is maximum at $V = \frac{V_0}{2}$.

Therefore, maximum value of T is $\frac{P_0V_0}{4R}$.

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CHAPTER 4

DPP 4.1

Subjective Type

1. During the warming process, $P = \left(\frac{P_i}{V_i}\right)V$

(a) $W = -\int_i^f P\,dV = -\int_{V_i}^{3V_i}\left(\frac{P_i}{V_i}\right)V\,dV$

$$W = -\left(\frac{P_i}{V_i}\right)\frac{V^2}{2}\Big|_{V_i}^{3V_i} = -\frac{P_i}{2V_i}(9V_i^2 - V_i^2) = -4P_iV_i$$

(b) $PV = nRT$ gives

$$\left[\left(\frac{P_i}{V_i}\right)V\right]V = nRT \Rightarrow T = \left(\frac{P_i}{nRV_i}\right)V^2$$

It is proportional to the square of the volume, according to $T = (P_i/nRV_i)V^2$

2. Work done by the gas

$$W = \int_i^f PdV$$

$$= \int_i^f \alpha V^2 dV = \frac{1}{3}\alpha(V_f^3 - V_i^3)$$

$V_f = 2V_i = 2(1.00\ \text{m}^3) = 2.00\ \text{m}^3$

$$W = \frac{1}{3}\left[(5.00\ \text{atm/m}^6)(1.013\times10^5\ \text{Pa/atm})\right] \times[(2.00\ \text{m}^3)^3 - (1.00\ \text{m}^3)^3]$$

$= 1.18$ MJ

Single Correct Answer Type

3. (d) Process AB is isochoric $\therefore W_{AB} = P\,\Delta V = 0$

Process BC is isothermal $\therefore W_{BC} = RT_2 \cdot \ln\left(\frac{V_2}{V_1}\right)$

Process CA is isobaric

$\therefore W_{CA} = -P\Delta V = -R\Delta T = -R(T_1 - T_2) = R(T_2 - T_1)$

(Negative sign is taken because of compression)

4. (d) $W_{BCOB} = -$ Area of triangle $BCO = -\frac{P_0V_0}{2}$

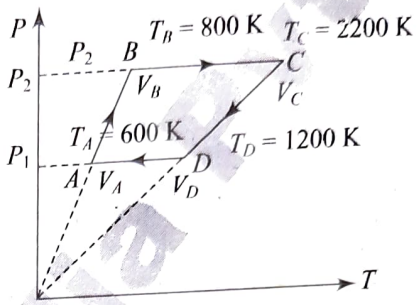
$W_{AODA} = +$ Area of triangle $AOD = +\frac{P_0V_0}{2}$

5. (a) As the volume is continuously increasing and the work of expansion is always positive, so the work done by the system continuously increases.

6. (c) Processes A to B and C to D are parts of straight line graphs of the form $y = mx$

Also $P = \frac{mR}{V}T \quad (\mu = 6)$

$\Rightarrow P \propto T$. So volume remains constant for the graphs AB and CD



So no work is done during processes for A to B and C to D i.e.,

$W_{AB} = W_{CD} = 0$ and $W_{BC} = P_2(V_C - V_B) = \mu R\,(T_C - T_B)$

$= 6R(2200 - 800) = 6R \times 1400$ J

Also $W_{DA} = P_1(V_A - V_D) = \mu R(T_A - T_B)$

$= 6R(600 - 1200) = -6R \times 600$ J

Hence work done in complete cycle

$W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$

$= 0 + 6R \times 1400 + 0 - 6R \times 600$

$= 6R \times 900 = 6 \times 8.3 \times 800 \approx 40$ kJ

7. (c) $W_{AB} = -P_0V_0$, $W_{BC} = 0$ and WCD $= 4P_0V_0$

$\Rightarrow \quad W_{ABCD} = -P_0V_0 + 0 + 4P_0V_0 = 3P_0V_0$

8. (b) $W = \int PdV = \int \frac{RT}{V}dV$

Since $V = kT^{2/3} \Rightarrow dV = \frac{2}{3}KT^{-1/3}dT$

Eliminating K, we find $\frac{dV}{V} = \frac{2}{3}\frac{dT}{T}$

Hence $W = \int_{T_1}^{T_2}\frac{2}{3}\frac{RT}{T}dT = \frac{2}{3}R(T_2 - T_1) = \frac{2}{3}R(30) = 20R$

9. (d) Before heating let the pressure of gas be P_1 from the equilibrium piston, $PA = kx_1$

$$\Rightarrow \quad x_1 = \frac{PA}{K}\left(\frac{nRT}{V}\right)\frac{A}{K} = \frac{1\times8.3\times100\times10^{-2}}{0.83\times100} = 0.1\ \text{m}$$

Since during heating process,

The spring is compressed further by 0.1 m

$\therefore \quad x_2 = 0.2$ m

Work done by gas

$$\frac{1}{2}\cdot100(0.2^2 - 0.1^2) = \frac{1}{2}100\cdot(0.1)(0.3) = 1.5\,\text{J}$$

10. (a) Work done by atmosphere $= P_{\text{rms}}\,\Delta V$

$$W_{\text{atm}} = P_{\text{atm}}\frac{V}{2} \qquad \text{(i)}$$

As; initially gas in container is in thermodynamic equilibrium with its surroundings.

$\therefore$ Pressure inside cylinder $= P_{\text{atm}}$ and $PV = nRT$

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$\Rightarrow \quad P_{atm} = nRT \quad$ or $\quad V = \dfrac{nRT}{P_{atm}}$

Putting in (i), $W = \dfrac{nRT}{2}$

11. (c) $W_{AB} = -\left(P_0V_0 + \dfrac{P_0V_0}{2}\right) = -\dfrac{3}{2}P_0V_0$

$W_{BC} = (2P_0)(2V_0) + \dfrac{P_0(2V_0)}{2} = 5P_0V_0$

$W_{ABC} = \dfrac{7}{2}P_0V_0$

Fill in the Blanks Type

12. $W = \mu RT \log_e \dfrac{V_2}{V_1}$

$= \left(\dfrac{m}{M}\right) RT \log_e \dfrac{V_2}{V_1} = 2.3 \times \dfrac{m}{M} RT \log_{10} \dfrac{V_2}{V_1}$

$= 2.3 \times \dfrac{96}{32} R\,(273+27) \log_{10} \dfrac{140}{70} = 2.3 \times 900R \log_{10} 2$

13. Amount of heat given = 540 calories
Change in volume ΔV = 1670 cc
Atmospheric pressure $P = 1.01 \times 10^6$ dyne/cm^2
Work done against atmospheric pressure

$W = P\Delta V = \dfrac{1.01 \times 10^6 \times 1670}{4.2 \times 10^7} \approx 40$ cal

14. Work done = Area of PV graph (here trapezium)

$= \dfrac{1}{2}(1 \times 105 + 5 \times 10^5) \times (5-1) = 12 \times 10^5$ J

15. Work done = Area enclosed by indicator diagram

$= \dfrac{1}{2} \times (3V - V)\,(4P - P) = 3PV$

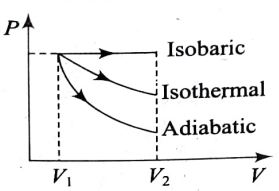
DPP 4.2

Single Correct Answer Type

1. (a) According to the first law of thermodynamics,
$\Delta Q = \Delta U + \Delta W$
In adiabatic process, $\Delta Q = 0$, hence $\Delta U = -\Delta W$.

2. (c) When heat is supplied at constant pressure, a part of it goes in the expansion of gas and remaining part is used to increase the temperature of the gas which in turn increases the internal energy.

3. (a) In thermodynamic process, work done is equal to the area covered by the PV curve with volume axis.
Hence, according to graph shown
$W_{adiabatic} < W_{isothermal} < W_{isobaric}$



4. (d) Fraction of supplied energy which increases the internal energy is given by

$f = \dfrac{\Delta U}{(\Delta Q)_P} = \dfrac{(\Delta Q)_V}{(\Delta Q)_P} = \dfrac{\mu C_V \Delta T}{\mu C_P \Delta T} = \dfrac{1}{\gamma}$

For diatomic gas $\gamma = \dfrac{7}{5} \Rightarrow f = \dfrac{5}{7}$

5. (c) $dU = C_V dT = \left(\dfrac{5}{2}R\right)dT$ or $dT = \dfrac{2(dU)}{5R}$ (i)

From first law of thermodynamics,

$dU = dQ - dW = Q - \dfrac{Q}{4} = \dfrac{3Q}{4}$

Now molar heat capacity, $C = \dfrac{dQ}{dT} = \dfrac{Q}{\dfrac{2(dU)}{5R}} = \dfrac{5RQ}{2\left(\dfrac{3Q}{4}\right)} = \dfrac{10}{3}R$

6. (b) $Q = \Delta U = U_f - U_i$ = [internal energy of 4 moles of a monoatomic gas + internal energy of 2 moles of a diatomic gas] – [internal energy of 4 moles of a diatomic gas]

$= \left(4 \times \dfrac{3}{2}RT + 2 \times \dfrac{5}{2}RT\right) - \left(4 \times \dfrac{5}{2}RT\right) = RT$

7. (b) $\Delta U = \mu C_V \Delta T = \dfrac{m}{M} C_V \Delta T = \dfrac{N}{N_A} C_V \Delta T$

$\Rightarrow \quad (\Delta U)_N = \dfrac{56 \times 10^3}{14} \times \dfrac{5}{2} R \times 300$

and $(\Delta U)_A = \dfrac{6 \times 10^{26}}{6 \times 10^{23}} \times \dfrac{3}{2} R \times 900 \Rightarrow (\Delta U)_N > (\Delta U)_A$

8. (d) $\Delta Q + \Delta U + \Delta W$; ΔU does not depend upon path.
$\because \quad \Delta W_A > \Delta W_B \Rightarrow \Delta Q_A > \Delta Q_B$

9. (d) ΔU remains same for both path
For path iaf: $\Delta U = \Delta Q - \Delta W = 50 - 20 = 30$ J
For path fi: $\Delta U = -30$ J and $\Delta W = -13$ J
$\Rightarrow \quad \Delta Q = -30 - 13 = -43$ J

10. (a) Heat given ΔQ = 40 J and Work done ΔW = 30 J
$\Rightarrow \quad \Delta U = \Delta Q - \Delta W = 40 - 30 = 10$ J

11. (b) Change in internal energy from $A \rightarrow B$ is

$\Delta U = \dfrac{f}{2}\mu R \Delta T = \dfrac{f}{2}(P_f V_f - P_i V_i)$

$= \dfrac{3}{2}(2P_0 \times 2V_0 - P_0 \times V_0) = \dfrac{9}{2}P_0V_0$

Work done in process $A \rightarrow B$ is equal to the area covered by the graph with volume axis i.e.,

$W_{A \rightarrow B} = \dfrac{1}{2}(P_0 + 2P_0) \times (2V_0 - V_0) = \dfrac{3}{2}P_0V_0$

Hence, $\Delta Q = \Delta U + \Delta W = \dfrac{9}{2}P_0V_0 + \dfrac{3}{2}P_0V_0 = 6P_0V_0$

12. (b) V-T graph is a straight line passing through origin. Hence, $V \propto T$ or P = constant
$\therefore \quad \Delta Q = nC_p \Delta T$ and $\Delta U = nC_v \Delta T$
Also $\Delta W = \Delta Q - \Delta U = \mu(C_P - C_V)\Delta T$

$\therefore \quad \dfrac{\Delta Q}{\Delta W} = \dfrac{nC_P \Delta T}{n(C_P - C_V)\Delta T} = \dfrac{C_P}{C_P - C_V} = \dfrac{1}{1 - \dfrac{C_V}{C_P}}$

$\dfrac{C_V}{C_P} = \dfrac{3}{5}$ for helium gas. Hence $\dfrac{\Delta Q}{\Delta W} = \dfrac{1}{1 - 3/5} = \dfrac{5}{2}$

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13. (c) $\Delta Q = \mu C_P \Delta T = \frac{7}{2}\mu R\Delta T \quad \left(C_P = \frac{7}{2}R\right)$

$\Delta U = \mu C_V \Delta T = \frac{5}{2}\mu R\Delta T \quad \left(C_V = \frac{5}{2}R\right)$

and $\Delta W = \Delta Q - \Delta U = \mu R\Delta T$

$\Rightarrow \Delta Q : \Delta U : \Delta W = 7 : 5 : 2$

14. (b) Since the gas is enclosed in a vessel, therefore, during heating process, volume of the gas remains constant. Hence, no work is done by the gas. It means heat supplied to the gas is used to increase its internal energy only.

Initial internal energy of the gas is $U_1 = N\left(\frac{5}{2}R\right)T$

Since n moles get dissociated into atoms, therefore, after heating, vessel contains $(N - n)$ moles of diatomic gas and $2n$ moles of a mono-atomic gas. Hence, the internal energy for the gas, after heating, will be equal to

$$U_2 = (N-n)\left(\frac{5}{2}R\right)T + 2n\left(\frac{3}{2}R\right)T = \frac{5}{2}NRT + \frac{1}{2}nRT$$

Hence, heat supplied = Increase in internal energy

$$= (U_2 - U_1) = \frac{1}{2}nRT$$

Fill in the Blanks Type

15. $\Delta Q + \Delta U + \Delta W = (U_f - U_i) + \Delta W$

$\Rightarrow 30 - (U_f - 40) + 10 \Rightarrow U_t = 60$ J

16. In first process using $\Delta Q = \Delta U + \Delta W$

$\Rightarrow 8 \times 10^5 = \Delta U + 6.5 \times 10^5 \Rightarrow \Delta U = 1.5 \times 10$ J

Since final and initial states are same in both process

So ΔU will be same in both process

For second process using $\Delta Q = \Delta U + \Delta W$

$\Rightarrow 10^5 = 1.5 \times 10^5 + \Delta W \Rightarrow \Delta W = -0.5 \times 10^5$ J

17. In isothermal process temperature remains constant, i.e., $\Delta T = 0$. Hence according to $C = \frac{Q}{m\Delta T} \Rightarrow C_{iso} = \infty$

18. This is the case of free expansion of gas. In free expansion, $\Delta U = 0 \Rightarrow$ Temp. remains same.

19. In isothermal compression, there is always an increase of heat which must flow out the gas.

$\Delta Q = \Delta U + \Delta W \quad \Rightarrow \quad \Delta Q = \Delta W \ (\because \Delta U = 0)$

$\Rightarrow \Delta Q = -1.5 \times 10^4 \text{ J} = \frac{1.5 \times 10^4}{4.18}$ cal $= -3.6 \times 10^3$ cal

20. In case of adiabatic expansion ΔW = positive and $\Delta Q = 0$ from FLOT $\Delta Q = \Delta U + \Delta W \Rightarrow \Delta U = -\Delta W$, i.e., ΔU will be negative.

21. $PV^\gamma = \text{constant} \Rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma \Rightarrow \frac{P_2}{1} = \left(\frac{V_1}{V_1/4}\right)^{3/2} = 8$

$\Rightarrow P_2 = 8$ atm

22. $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \Rightarrow T_2 = 300\left(\frac{27}{8}\right)^{\frac{5}{3}-1} = 300\left(\frac{27}{8}\right)^{\frac{2}{3}}$

$$= 300\left\{\left(\frac{27}{8}\right)^{1/3}\right\}^2 = 800\left(\frac{3}{2}\right)^2 = 675 \text{ K}$$

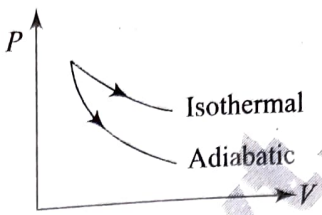
$\Rightarrow \Delta T = 675 - 300 = 375$ K

23. In thermodynamic processes.

Work done = Area covered by PV diagram with V-axis

From graph it is clear that $(\text{Area})_{iso} > (\text{Area})_{adi}$

$\Rightarrow W_{iso} > W_{adi}$



24. $dQ = 0 = -2 + dW \Rightarrow dW = 2$ J

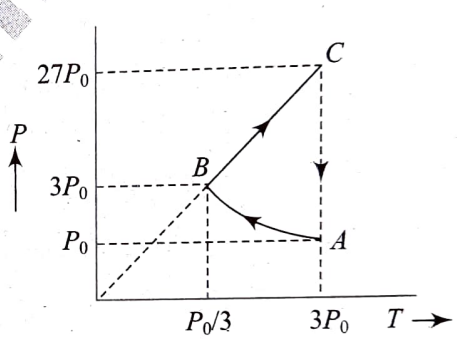
$\Rightarrow$ Work done by the gas = 2 J

$\Rightarrow$ Work done on the gas = –2 J

DPP 4.3

Subjective Type

1. (i)



(ii) Work done in process AB

$$\Delta w_{AB} = \frac{2r\left[\frac{T_0}{3} - 3T_0\right]}{\left(\frac{1}{3} - 1\right)}$$

$w = -8RT_0$

$\Delta Q_{AB} = du + \Delta w = -nq\Delta T - 8RT_0$

$\Delta Q_{AB} = -16RT_0$ (ii)

for process $B \rightarrow C$, $W_{BC} = 0$

$du = 8RT_0 \qquad \Delta Q_{BC} = 8RT_0$

and $C \rightarrow A$ (isothermal process)

$$\Delta W_{CA} = 2R(3T_0)\ln\frac{V_\varepsilon}{V_i} \quad \left(\frac{V_\varepsilon}{V_i} = 27\right)$$

$\Delta W_{CA} = 18RT_0 \ln 3$ and $\Delta Q_{CA} = 18RT_0 \ln 3$

New work done

$\Delta W = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CA}$

$= -8RT_0 + 0 + 18RT_0 \ln 3$

$\Delta W = RT_0(18 \ln 3 - 8)$

and $\Delta Q_{net} = \Delta Q_{BC} + \Delta Q_{CA} = -8RT_0 + 18RT_0 \ln 3$

$\Delta Q_{net} = RT_0(18 \ln 3 + 8)$

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Single Correct Answer Type

2. (a) $\Delta E_{int} = 0$, for a complete cycle and for given cycle work done is negative, so from first law of thermodynamics Q will be negative i.e. $Q < 0$.

3. (a) $\Delta W_{AB} = 0$ as V = constant

$\therefore \quad \Delta Q_{AB} = \Delta U_{AB} = 50$ J

$U_A = 1500$ J $\therefore$ $U_B = (1500 + 50)$J $= 1550$ J (Given)

$\Delta W_{BC} = -\Delta U_{BC} = -40$ J

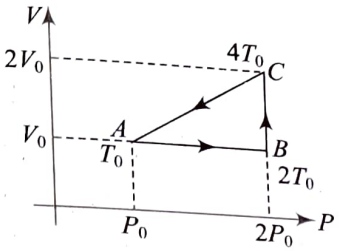
$\therefore \quad \Delta U_{BC} = 40$ J $\therefore$ $U_C = (1550 + 40)$ J $= 1590$ J (Given)

4. (d) $W = \frac{1}{2}P_0V_0 = \frac{1}{2}P_0V_0$

Heat supplied $= Q_{AB} + Q_{BC} = C_V T_0 + C_P 2T_0 = \frac{13}{2}RT_0$

$\therefore$ Efficiency $= \dfrac{\frac{1}{2}P_0V_0}{4P_0V_0} \times 100 \qquad \left(\because \frac{13}{2}P_0V_0 = \frac{13}{2}RT_0\right)$

$= \frac{1}{13} \times 100 = 7.7\%$



5. (a) Work done by gas in all four process is positive and in order

$W_A > W_B > W_C > W_D$

$\Rightarrow$ (c) is false

The change in initial energy U is same for all process.

$\therefore \quad Q_A = U + W_A$ (1)

$Q_B = U + W_B$ (2)

$Q_C = U + W_C$ (3)

$Q_D = U + W_D$ (4)

Hence $Q_A > Q_B > Q_C > Q_D$

$\Rightarrow$ (d) is false

from equation (1) and (4)

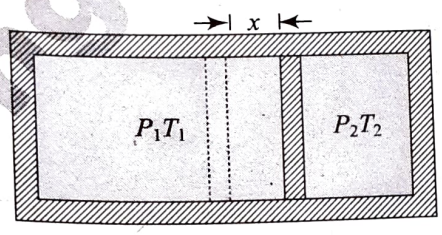
$Q_A - Q_D = W_A - W_D$

$\Rightarrow$ (a) is true

From equation (2) and (3) $Q_B - Q_C = W_B - W_C$

$\Rightarrow$ (b) is false.

6. (c) As finally the piston is in equilibrium, both the gases must be at same pressure P_f. It is given that displacement of piston be in final state x and if A is the area of cross-section of the piston. Hence, the final volumes of the left and right part finally can be given by figure as



$$V_L = \frac{V_0}{2} + Ax \text{ and } V_R = \frac{V_0}{2} - Ax$$

As it is given that the container walls and the piston are adiabatic in left side and the gas undergoes adiabatic expansion and on the right side the gas undergoes adiabatic compressive. Thus, we have for initial and final state of gas on left side

$$P_1\left(\frac{V_0}{2}\right)^\gamma = P_f\left(\frac{V_0}{2} + Ax\right)^\gamma \quad \text{(i)}$$

Similarly for gas in right side, we have

$$P_2\left(\frac{V_0}{2}\right)^\gamma = P_f\left(\frac{V_0}{2} - Ax\right)^\gamma \quad \text{(ii)}$$

From eq. (i) and (ii)

$$\frac{P_1}{P_2} = \frac{\left(\frac{V_0}{2} + Ax\right)^\gamma}{\left(\frac{V_0}{2} - Ax\right)^\gamma} \Rightarrow Ax = \frac{V_0}{2}\frac{[P_1^{1/\gamma} - P_2^{1/\gamma}]}{[P_1^{1/\gamma} + P_2^{1/\gamma}]}$$

Now from equation (i) $P_f = \dfrac{P_1\left(\frac{V_0}{2}\right)^\gamma}{\left(\frac{V_0}{2} + Ax\right)^\gamma}$

7. (a) For adiabatic process, $T_1V_b^{\gamma-1}$ = Constant

For bc curve, $T_1V_b^{\gamma-1} = T_2V_c^{\gamma-1}$ or $\frac{T_2}{T_1} = \left(\frac{V_b}{V_c}\right)^{\gamma-1}$ (i)

For ad curve, $T_1V_a^{\gamma-1} = T_2V_d^{\gamma-1}$ or $\frac{T_2}{T_1} = \left(\frac{V_a}{V_d}\right)^{\gamma-1}$ (ii)

From equations (i) and (ii), $\frac{V_b}{V_c} = \frac{V_a}{V_d}$

Multiple Correct Answers Type

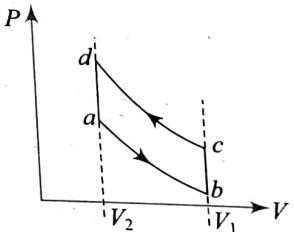
8. (a, b, d)

$$\rho = \frac{\rho}{M_0}RT \Rightarrow \frac{P}{\rho} = \frac{R}{M_0}T$$

Slope of the curve a Temperature

Hence, cd and ab are isothermal processes.

$$\rho \propto \frac{1}{V}$$



Equivalent PV diagram

i.e. bc and da are constant volume process

(a) and (b) are true.

Temp. in cd process is greater than ab.

Net work done by the gas in the cycle is negative, as is clear in the PV-diagram.

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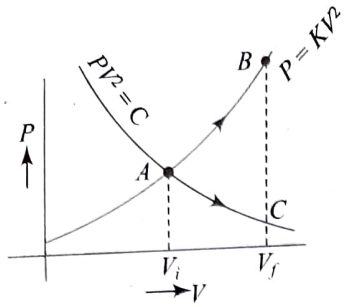
9. (b, d)

From equation $PV = nRT$



$P_C < P_B$, $V_C = V_B$ and $T_B > T_C$

$W_{AB} > W_{AC}$ $(T_B - T_A) > T_C - T_A$

So by Ist law

$Q = \Delta U + W$

$Q = \frac{1}{2} nR\Delta T + W$

$Q_{AB} > Q_{AC}$

Comprehension Type

10. (a) $n = 2,\ C_V = \frac{3}{2}R, C_p = \frac{5}{2}R$

$T_A = 300$ K

$V_A = V_0,\ V_B = 2V_0$

$V_D = V_C = 4V_0$

For process $A \rightarrow b$, $V \propto T$

$\frac{T_B}{T_A} = \frac{V_B}{V_A}$

$T_B = T_A\left(\frac{V_B}{V_A}\right) = 300 \times 2 = 600$ K

11. (b) $V \propto T$ and P = constant

$Q_{AB} = nC_p dT = nC_p(T_B - T_A)$

$= 2\left(\frac{5}{2}R\right)(600 - 300)$

$= 1500R$ (absorbed)

12. (c) T = constant, $dU = 0$

$Q_{BC} = W_{BC} = nRT_B \ln\left(\frac{V_C}{V_B}\right)$

$= 2R(600)\ln\left(\frac{4V_0}{2V_0}\right)$

$= 1200R \ln(2) \cong 832R$ (absorbed)

13. (b) V = constant

$Q_{CD} = nC_v dT = nC_v(T_D - T_C)$

$= n\left(\frac{3}{2}R\right)(T_A - T_B)$

(since $T_D = T_A$ and $T_C = T_B$)

$= -900R = 900R$ (released)

14. (d) $dU = 0$ as T = constant

$Q_{DA} = W_{DA} = nRT_D \ln\left(\frac{V_A}{V_D}\right)$

$= 600R \ln(1/4)$

$= -832R = 832R$ (released)

15. (b) $W_{net} = Q_{AB} + Q_{BC} + Q_{CD} + Q_{DA}$

$= 600R$ (since $dU = 0$)

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CHAPTER 5

DPP 5.1

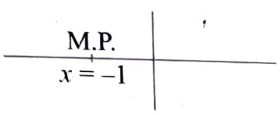
Subjective Type

1. (a) Given $F = 3x + 3$ (i)

We find the mean position at which net force on the particle is zero.

$\Rightarrow \quad 3x + 3 = 0 \quad \Rightarrow \quad x = -1$

If we put $x = 0$ in eq. (i), then



$F = 3N$ (away from M.P.) (a)

Now put $x = -2$ in eq. (i)

$F = -3$ N (away from M.P.) (b)

(a) and (b) we conclude that particle doesn't perform SHM.

(b) Given $F = -3x - 3$ (i)

at M.P. $F = 0$

$\Rightarrow \quad x = -1$

Now put $x = 0$ in eq. (i)

$\Rightarrow \quad F = -3$ N (towards M.P.)

If $\quad x = -2$, $F = 3$ N (towards M.P.)

We conclude from the above calculation that in every case (whether the particle is left from M.P. or right from M.P.) force acts towards M.P. so the particle performs SHM.

(c) Given $F = -3x + 3$

When $F = 0$, $x = 1$ (M.P.)

Now put $x = 0$

Then $F = 3$N (towards M.P.)

If $x = 2$, $F = -3$N (towards M.P.)

i.e. particle performs SHM.

(d) Given $F = 3x - 3$

Mean position at $x = 1$.

When $x = 0$; $F = -3$N (away from M.P.)

$x = 2$; $F = 3$ N (away from M.P.)

Particle doesn't perform SHM.

Single Correct Answer Type

2. (b) Body collides elastically with walls of room. So, there will be no loss in its energy and it will remain colliding with walls of room, so its motion will be periodic.

There is no change in energy of the body, hence there is no acceleration, so its motion is not SHM.

3. (c) As the given equation is

$x = a \cos (at)^2$

is a cosine function. Hence, it is an oscillatory motion.

Now, putting $t + T$ in place of t

$x(t + T) = a \cos [\alpha(t + T)]^2 \qquad [\because x(t) = a \cos (\alpha t)^2]$

$= a \cos [\alpha t^2 + \alpha T^2 + 2\alpha tT] \neq x(t)$

where T is supposed as period of the function $w(t)$.

Hence, it is not periodic.

4. (d) Standard equation of S.H.M. $\frac{d^2y}{dt^2} = -\omega^2 y$ is not satisfied by $y = a \tan \omega t$

which will be periodic.

Clearly, this equation represents SHM.

5. (b) Given equation of motion is

$y = \sin^3 \omega t$

$(3 \sin \omega t - 4 \sin 3\omega t)/4$

$[\because \sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta]$

$$\Rightarrow \frac{dy}{dt} = \left[\frac{d}{dt}(3 \sin \omega t) - \frac{d}{dt}(4 \sin 3\omega t)\right]/4$$

$$\Rightarrow 4\frac{dy}{dt} = 3\omega \cos \omega t - 4 \times [3\omega \cos 3\omega t]$$

$$\Rightarrow 4 \times \frac{d^2y}{dt^2} = -3\omega^2 \sin \omega t + 12\omega \sin 3\omega t$$

$$\Rightarrow \frac{d^2y}{dt^2} = -\frac{3\omega^2 \sin \omega t + 12\omega^2 \sin 3\omega t}{4}$$

$\Rightarrow \frac{d^2y}{dt^2}$ is not proportional to y

Hence, motion is not SHM.

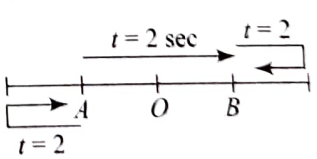
6. (d) $|A| = \omega^2 x \Rightarrow \frac{|A|}{x} = \omega^2$

From the given value $\frac{|A|}{x} = \omega^2 = 4 \Rightarrow \omega = 2$

Also $\omega = \frac{2\pi}{T} \Rightarrow 2 = \frac{2\pi}{T} \Rightarrow T = \pi$ sec

7. (d) From the given information it can be inferred that points A and B are equidistant from mean position

Hence, from diagram it is clear that time period of oscillation $= 2 + 2 \times 2 + 2 = 8$ s.



8. (b) Time taken by particle to move from $x = 0$ (mean position) t

$x = 4$ (extreme position) $= \frac{T}{4} = \frac{1.2}{4} = 0.3$ s

Let t be the time taken by the particle to move from $x = 0$

$x = 2$ cm

$y = a \sin \omega t \Rightarrow 2 = 4 \sin \frac{2\pi}{T} t \Rightarrow \frac{1}{2} = \sin \frac{2\pi}{1.2} t$

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$\Rightarrow \frac{\pi}{6}=\frac{2\pi}{1.2}t \Rightarrow t=0.1$ s. Hence time to move from $x = 2$ to $x = 4$ will be equal to $0.3 - 0.1 = 0.2$ s

Hence total time to move from $x = 2$ to $x = 4$ and back again $= 2 \times 0.2 = 0.4$ sec

9. (b) $x_1 = a \sin(\omega t + \phi_1)$

$x_2 = a \sin(\omega t + \phi_2)$

$$\Rightarrow \quad |x_1 - x_2| = 2a \sin\left(\omega t + \frac{\phi_1+\phi_2}{2}\right)\cos\left(\frac{\phi_1-\phi_2}{2}\right)$$

To maximize $|x_1 - x_2|$: $\sin\left(\omega t + \frac{\phi_1+\phi_2}{2}\right)=1$

$$\Rightarrow \quad a\sqrt{2} = 2a \times 1 \times \cos\left(\frac{\phi_1-\phi_2}{2}\right)$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \cos\left(\frac{\phi_1-\phi_2}{2}\right)$$

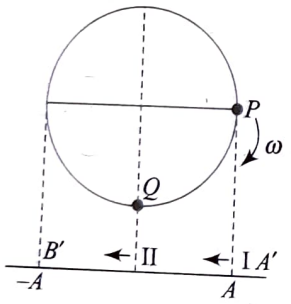
$$\Rightarrow \quad \frac{\pi}{4}=\frac{\phi_1-\phi_2}{2} \quad \Rightarrow \quad \phi_1-\phi_2=\frac{\pi}{2}$$

10. (c) Equation of motion $y = a \cos \omega t$

$$\Rightarrow \quad \frac{a}{2} = a\cos\omega t \Rightarrow \cos\omega t = \frac{1}{2} \Rightarrow \omega t = \frac{\pi}{3}$$

$$\Rightarrow \quad \frac{2\pi t}{T}=\frac{\pi}{3} \Rightarrow t=\frac{\frac{\pi}{3}\times T}{2\pi}=\frac{4}{3\times 2}=\frac{2}{3}\text{sec}$$

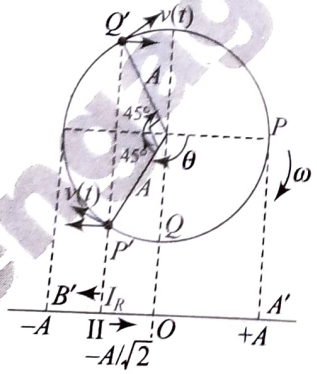
11. (b) This problem is easy to solve with the help of phasor diagram. First we draw the initial position of both the particles on the phasor as shown in figure.



at $t = 0$

From above figure, phase difference between both the particles is $\pi/2$. They will cross each other when their projection from the circle on the horizontal diameter meet at one point.

Let after time t, both will reach at $P'Q'$ point having phase difference $\pi/2$ as shown in figure.



After $t = t$

Both will meet at $-A/\sqrt{2}$

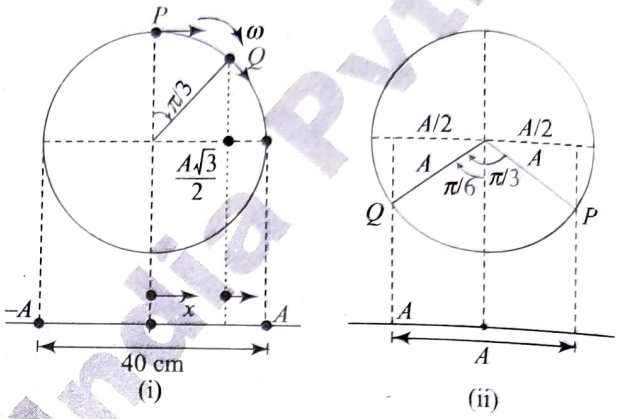
When they meet, angular displacement of P is

$$\theta = \frac{\pi}{2}+\frac{\pi}{4}=\frac{3\pi}{4}$$

So they will meet after time, $t = \frac{3\pi}{4\times\omega}$

$$t = \frac{3\pi}{4\times 2\pi}\times T = \frac{3T}{8}\text{sec}$$

12. (a) Let us assume that one particle starts from mean position and another starts at a distance x having $\phi = \pi/3$. This condition is shown in figure.



The above figure shows the situation of maximum distance between them.

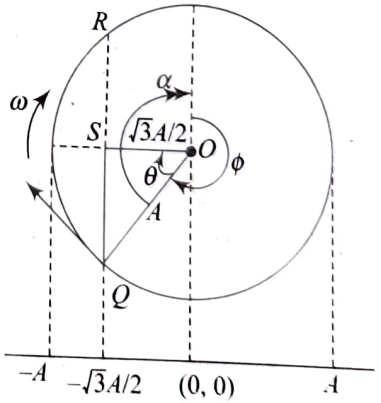
So maximum distance $= A = 10$ cm (as $2A = 20$ cm)

Multiple Correct Answers Type

13. (a,b,c)

Figure shows the solution of the problem with the help of phasor. Horizontal component of velocity at Q gives the required direction of velocity at $t = 0$.

In ΔOSQ $\cos\theta = \frac{\sqrt{3}/2A}{A}=\frac{\sqrt{3}}{2} \Rightarrow \theta=\frac{\pi}{6}$



Now $\varphi = \frac{3\pi}{2}-\frac{\pi}{6}=\frac{8\pi}{6}=\frac{4\pi}{3}$

So equation of SHM is $x = A\sin\left(\omega t + \frac{4\pi}{3}\right)$

Now to reach the particle at left extreme point it will travel angle θ along the circle. So time taken.

$$t = \frac{\theta}{\omega}=\frac{\pi}{6\omega} \quad \Rightarrow \quad t = \frac{T}{12}\text{sec}$$

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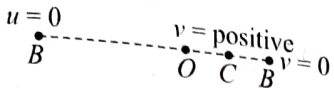
To reach the particle at mean position, it will travel an angle

$$\alpha = \frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}$$

So, time taken = $\frac{\alpha}{\omega} = \frac{T}{3}$ sec

14. (a, c, d)

Consider the diagram.



(a) When the particle is 3 cm away from A going towards B, velocity is towards AB, i.e., positive.
In SHM, acceleration is always towards mean position (O) in this case. Hence, it is positive.

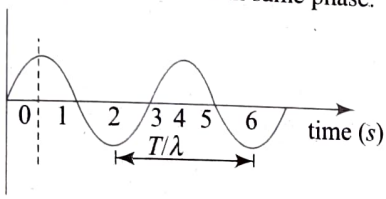
(b) When the particle is at C, velocity is towards B, hence positive.

(c) When the particle is 4 cm away from B going towards A, velocity is negative and acceleration is towards mean position (O) hence negative.

(d) Acceleration is always towards mean position (O). When the particle is at B acceleration and force are towards BA that is negative.

15. (b, d)

It is clear from the curve that points corresponding to $t = 2$ s and $t = 6$ s are separated by a distance belongs to one time period. Hence, these points must be in same phase.



Similarly, points belong to $t = 1$ s and $t = 5$ s are at separation of one time period, hence must be in phase.

DPP 5.2

Subjective Type

1. $U = (x^2 - 4x + 3)$

$$F = -\frac{du}{dx} = -2x + 4 \Rightarrow F = -2(x-2)$$

(a) For equilibrium, $F = 0 \Rightarrow F = 2$

(b) $\omega = \sqrt{\frac{2}{1}} = \sqrt{2} \Rightarrow T = \frac{2\pi}{\sqrt{2}} = \sqrt{2}\pi$

(c) $\frac{1}{2}KA^2 = \frac{1}{2}mv^2 \Rightarrow A = 2\sqrt{3}$

Single Correct Answer Type

2. (c) $2A_i\omega_i^2 = A_f\omega_f^2$

$A_i\omega_i = A_f\omega_f$

If $\omega_f = 2\omega_i$ and $A_f = \frac{A_i}{2}$,

The condition will be satisfied.

3. (b) $x = a\cos(\omega t + \theta)$ (i)

and $v = \frac{dx}{dt} = -a\omega\sin(\omega t + \theta)$ (ii)

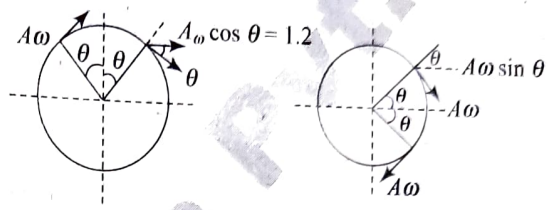
Given at $t = 0$, $x = 1$ cm and $v = \pi$ and $\omega = \pi$

Putting these values in equation (i) and (ii) we will get

$\sin\theta = \frac{-1}{a}$ and $\cos\theta = \frac{1}{A}$

$$\Rightarrow \sin^2\theta + \cos^2\theta = \left(-\frac{1}{a}\right)^2 + \left(\frac{1}{a}\right)^2 \Rightarrow a = \sqrt{2}\text{ cm}$$

4. (d)



$A\omega\cos\theta = 1.2$ $\qquad A\omega\sin\theta = 1.6$

$\tan\theta = 4/3 \Rightarrow \theta = 53°$

$\therefore \quad A\omega \cdot \frac{3}{5} = 1.2 \Rightarrow A\omega = 2$ m/s

5. (c) $v_{max} = a\omega \Rightarrow \omega = \frac{v_{max}}{a} = \frac{10}{4}$

Now, $v = \omega\sqrt{a^2 - y^2} \Rightarrow v^2 = \omega^2(a^2 - y^2) \Rightarrow y^2 = a^2 - \frac{v^2}{\omega^2}$

$$\Rightarrow y = \sqrt{a^2 - \frac{v^2}{\omega^2}} = \sqrt{4^2 - \frac{5^2}{(10/4)^2}} = 2\sqrt{3}\text{ cm}$$

6. (d) Average velocity $\bar{v} = \frac{\int_0^t \frac{dx}{dt}\cdot dt}{t} = \frac{\int_0^t dx}{t} = \frac{x(t) - x(0)}{t}$

$$= \frac{A(\cos\pi/6 - 1)}{\pi/6\omega} = \frac{3A\omega}{\pi}(\sqrt{3} - 2)$$

Since particle does not change its direction in the given interval, average speed becomes

$$|\bar{v}| = \frac{3A\omega}{\pi}(2 - \sqrt{3})$$

7. (c) Total energy in SHM $E = \frac{1}{2}m\omega^2a^2$ (where a = amplitude)

Potential energy $U = \frac{1}{2}m\omega^2(a^2 - y^2) = E - \frac{1}{2}m\omega^2y^2$

When $y = \frac{a}{2} \Rightarrow U = E - \frac{1}{2}m\omega^2\left(\frac{a^2}{4}\right) = E - \frac{E}{4} = \frac{3E}{4}$

8. (b) $E_1 = \frac{1}{2}Kx^2 \Rightarrow x = \sqrt{\frac{2E_1}{K}}$, $E_2 = \frac{1}{2}Ky^2 \Rightarrow y = \sqrt{\frac{2E_2}{K}}$

and $E = \frac{1}{2}K(x + y)^2 \Rightarrow x + y = \sqrt{\frac{2E}{K}}$

$$\Rightarrow \sqrt{\frac{2E_1}{K}} + \sqrt{\frac{2E_2}{K}} = \sqrt{\frac{2E}{K}} \Rightarrow \sqrt{E_1} + \sqrt{E_2} = \sqrt{E}$$

9. (b) $E = \frac{1}{2}m\omega^2A^2 \Rightarrow E = \frac{1}{2}m(2\pi f)^2A^2 \Rightarrow A = \frac{1}{2\pi f}\sqrt{\frac{2E}{m}}$

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3. (b) The two springs on left side having spring constant of $2k$ each are in series; equivalent constant is $\frac{1}{\left(\frac{1}{2k}+\frac{1}{2k}\right)}=k$. The two springs on right hand side of mass M are in parallel. Their effective spring constant is $(k+2k)=3k$. Equivalent spring constants of value k and $3k$ are in parallel and their net value of spring constant of all the four springs is $k+3k=4k$.

$\therefore$ Frequency of mass is $n=\frac{1}{2\pi}\sqrt{\frac{4k}{M}}$

4. (b) $T_1=2\pi\sqrt{\frac{m}{K_1}}, T_2=2\pi\sqrt{\frac{m}{K_2}}, T=2\pi\sqrt{\frac{m}{K_{eq}}}$

$K_{eq}\frac{K_1K_2}{K_1+K_2}$ (for series), $t_1^2=\frac{2\pi m}{K_1}$

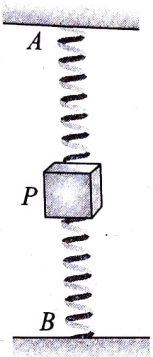
$t_2^2=\frac{2\pi m}{K_2}$ and $T^2=2\pi m\left(\frac{1}{K_1}+\frac{1}{K_2}\right)$

$\Rightarrow \quad T^2=t_1^2+t_2^2$

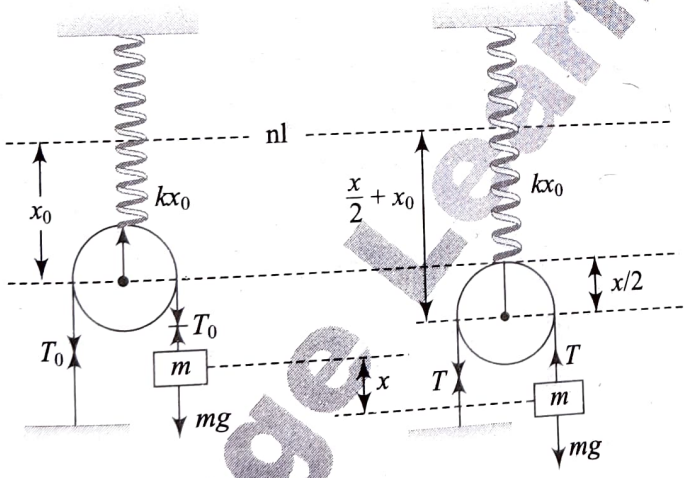
5. (b) $Mg=K\times 20-K\times 10 \Rightarrow K=\frac{mg}{10/100}$ N/m

$T=2\pi\sqrt{\frac{M}{K}}$

$=2\pi\sqrt{\frac{1}{49\times 4}}=\frac{\pi}{7}$ sec



6. (d) Let us assume that in equilibrium condition, spring is x_0 elongated from its natural length.



When equillibrium

When displaced block by x

In equilibrium, $T_0 = mg$

and $kx_0 = 2T_0$ (i)

$\Rightarrow \quad kx_0 = 2mg$

If the mass m moves down a distance x from its equilibrium position, then pulley will move down by $\frac{x}{2}$. So the extra force in spring will be $\frac{kx}{2}$. From figure,

$$F_{net}=mg-T=mg-\frac{k}{2}\left(x_0+\frac{x}{2}\right)$$

$$F_{net}=mg-\frac{kx_0}{2}-\frac{kx}{4}$$

From eq. (i)

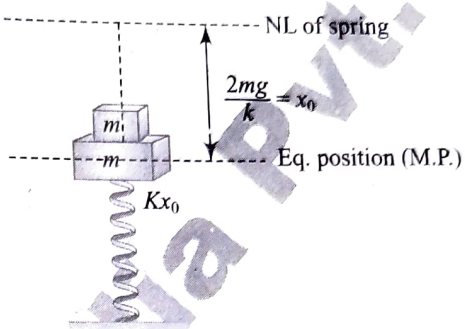
$$F_{net}=\frac{-kx}{4}$$

Now compare eq. (ii) with $F=-K_{SHM}x$, then (ii)

$$K_{SHM}=\frac{K}{4}$$

$$\Rightarrow \quad T=2\pi\sqrt{\frac{m}{K_{S.H.M}}}=2\pi\sqrt{\frac{4m}{K}}$$

7. (c)



The blocks will remain in contact till the blocks do not go above the natural length of the spring, because after this condition the deceleration of the lower block becomes more than the upper block due to spring force. So they will get separated.

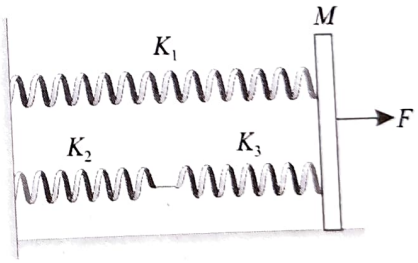
So, maximum possible amplitude = $x_0=\frac{2mg}{k}$

Multiple Correct Answers Type

8. (a, b)

$$K_{eq}=K_1+\frac{K_2K_3}{K_2+K_3} \because F=K_{eq}A$$

$$\Rightarrow \quad A=\frac{F}{K_{eq}}=\frac{(K_2+K_3)F}{K_1K_2+K_2K_3+K_3K_1}$$



$$f=\frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M}}=\frac{1}{2\pi}\left[\frac{K_1K_2+K_2K_3+K_3K_1}{M(K_2+K_3)}\right]^{1/2}$$

9. (b, c, d)

$$mg=Kx_0 \quad \Rightarrow \quad x_0=\frac{10}{500}=2 \text{ cm}$$

$$\omega=\sqrt{\frac{500}{1}}=10\sqrt{5}\ \frac{\text{rad}}{\text{sec}}$$

$\therefore$ Maximum velocity $= A\omega = 3\times 10^{-2}\times 10\sqrt{5}$

$= 30\sqrt{5}$ cm/sec

$\Rightarrow$ Maximum Acceleration $= A\omega^2 = 15$ m/s^2

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10. (a, b)

Given $A = 0.1$ m, $K = 100$ N/m

$m_1 = 1$ kg, $m_2 = 3$ kg

Energy of system is not conserved.

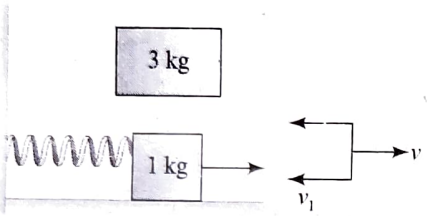
$$\frac{1}{2}KA^2 = \frac{1}{2}m_1v_1^2 \Rightarrow v_1 = \frac{KA^2}{m_1}$$

$v_1 = 1$ m/s

$$m_1v_1 = (m_1 + m_2)v \Rightarrow v = \frac{1}{(m_1 + m_2)}$$

$\Rightarrow$ $v = 0.25$ m/s

$\because$ Frequency $= \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi}\sqrt{\frac{100}{4}} = \frac{5}{2\pi}$ Hz

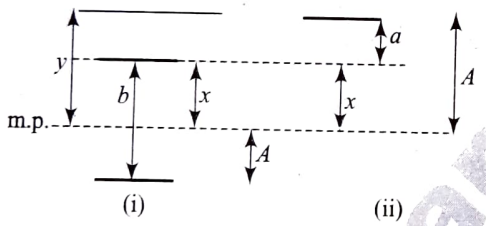


Now amplitude

$$\frac{1}{2}KA'^2 = \frac{1}{2}(m_1 + m_2)v^2 \Rightarrow A'^2 = \frac{(0.25)^2 \times 4}{100}$$

$\Rightarrow$ $A' = 5$ cm

11. (a, b)



(i) (ii)

From figure (i) $b = A + x$ (i)

From figure (ii) $A = a + x$ (ii)

From eq. (i) and (ii)

$b = a + 2x \Rightarrow 2x = b - a$

and $x = mg/k \Rightarrow K = \frac{2mg}{b - a}$

Oscillation frequency

$$= f = \frac{1}{2\pi}\sqrt{\frac{K}{m_{total}}} = \frac{1}{2\pi}\sqrt{\frac{2mg}{(b-a)(M+m)}}$$

12. (c, d)

By momentum conservation,

$5 \times 3 + 10 \times 2 = 7v$

$v = 5$ m/s

By energy conservation,

$$\frac{1}{2}\cdot 5.3^2 + \frac{1}{2}\times 2 \times (10)^2 = \frac{1}{2}\times 7 \times 5^2 + \frac{1}{2}Kx^2$$

which gives $x = 0.25$ m.

Time period of oscillation, $T = 2\pi\sqrt{\frac{\mu}{K}}$

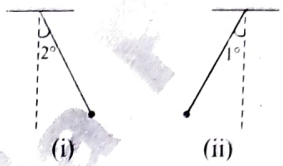
Here, $\mu = \left(\frac{m_1m_2}{m_1 + m_2}\right) = \frac{10}{7} \Rightarrow T = \pi/14$

First compression occurs $= \frac{3T}{4} = \frac{3\pi}{56}$ s

DPP 5.4

Subjective Type

1. Consider the situations shown in the diagram (i) and (ii)



(i) (ii)

Assuming the two pendulums follow the following functions of their angular displacements

$\theta_1 = \theta_0 \sin(\omega t + \phi_1)$ (i)

and $\theta_2 = \theta_0 \sin(\omega t + \phi_2)$ (ii)

As it is given that amplitude and time period being equal but phases being different.

Now, for first pendulum at any time t

$\theta_1 = +\theta_0$ [Right extreme]

From Eq. (i), we get

$\Rightarrow$ $\theta_0 = \theta_0 \sin(\omega t + \phi_1)$ or $1 = \sin(\omega t + \phi_1)$

$\Rightarrow$ $\sin\frac{\pi}{2} = \sin(\omega t + \phi_1)$

or $(\omega t + \phi_1) = \frac{\pi}{2}$ (iii)

Similarly, at the same instant t for pendulum second, we have

$$\theta_2 = -\frac{\theta_0}{2}$$

where $\theta_0 = 2°$ is the angular amplitude of first pendulum. For the second pendulum, the angular displacement is one degree, therefore, $\theta_2 = \frac{\theta_0}{2}$ and negative sign is taken to show for being left to mean position.

From Eq. (ii), then $-\frac{\theta_0}{2} = \theta_0 \sin(\omega t + \phi_2)$

$\Rightarrow$ $\sin(\omega t + \phi_2) = -\frac{1}{2} \Rightarrow (\omega t + \phi_2) = -\frac{\pi}{6}$ or $\frac{7\pi}{6}$

or $(\omega t + \phi_2) = -\frac{\pi}{6}$ or $\frac{7\pi}{6}$ (iv)

From Eqs. (iv) and (iii), the difference in phases

$$(\omega t + \phi_2) - (\omega t + \phi_1) = \frac{7\pi}{6} - \frac{\pi}{2} = \frac{7\pi - 3\pi}{6} = \frac{4\pi}{6}$$

or $(\phi_2 - \phi_1) = \frac{4\pi}{6} = \frac{2\pi}{3} = 120°$

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Single Correct Answer Type

2. (d) Time period of a pendulum doesn't depend upon mass, but it depends upon length (distance between point of suspension and centre of mass).
In first three cases, lengths are same. So $T = T_1 = T_2$, but in last case centre of mass lowers which in turn increases the length. So in this cases time period T_3 will be more than the other cases.

3. (b) The effective acceleration of a bob in water

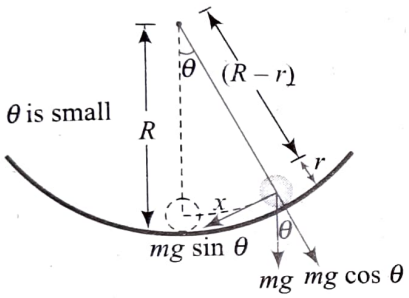
$= g' = g\left(1 - \frac{\sigma}{\rho}\right)$ where σ and ρ are the density of water and the bob, respectively. Since the period of oscillation of the bob in air and water are given as $T = 2\pi\sqrt{\frac{l}{g}}$ and $T' = 2\pi\sqrt{\frac{l}{g'}}$

$$\therefore \quad \frac{T}{T'} = \sqrt{\frac{g'}{g}} = \sqrt{\frac{g(1-\sigma/\rho)}{g}} = \sqrt{1-\frac{\sigma}{\rho}} = \sqrt{1-\frac{1}{\rho}}$$

Putting $\frac{T}{T'} = \frac{1}{\sqrt{2}}$, we obtain $\frac{1}{2} = 1 - \frac{1}{\rho} \Rightarrow \rho = 2$

4. (b) Tangential acceleration, $a_t = -g \sin\theta = -g\theta$

$$a_t = -g\frac{x}{(R-r)}$$



Motion is S.H.M., with time period

$$T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\frac{x}{\frac{gx}{(R-r)}}} = 2\pi\sqrt{\frac{R-r}{g}}$$

5. (c) If t is the time taken by pendulums to come in same phase again first time after $t = 0$;
N_S = Number of oscillations made by shorter length pendulum with time period T_S; and
N_L = Number of oscillations made by longer length pendulum with time period T_L, then

$$t = N_S T_S = N_L T_L$$

$$\Rightarrow \quad N_S 2\pi\sqrt{\frac{5}{g}} = N_L \times 2\pi\sqrt{\frac{20}{g}} \qquad \left(\because T = 2\pi\sqrt{\frac{l}{g}}\right)$$

$$\Rightarrow \quad N_S = 2N_L \text{ i.e. if } N_L = 1 \Rightarrow N_S = 2$$

6. (a) Tension in the string when bob passes through the lowest point

$$T = mg + \frac{mv^2}{r} = mg + mv\omega \qquad (\because v = r\omega)$$

putting $v = \sqrt{2gh}$ and $\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$

we get $T = m(g + \pi\sqrt{2gh})$

7. (b) Let T_1 and T_2 are the time period of the two pendulums

$T_1 = 2\pi\sqrt{\frac{100}{g}}$ and $T_2 = 2\pi\sqrt{\frac{121}{g}}$ $(T_1 < T_2$ because $l_1 < l_2)$

Let at $t = 0$, they start swinging together. Since their time periods are different, the swinging will not be in unision always. Only when number of completed oscillation differs by an integer, the two pendulum will again begin to swing together.
Let longer length pendulum complete n oscillation and shorter length pendulum complete $(n + 1)$ oscillation, for the unision swinging, then $(n+1)T_1 = nT_2$

$$(n+1) \times 2\pi\sqrt{\frac{100}{g}} = n \times 2\pi\sqrt{\frac{121}{g}} \Rightarrow n = 10$$

8. (b) $\frac{5T}{4} = T + \frac{T}{4}$

Bigger prendulum $(5T/4)$ Smaller pendulum (T)

By the time, the bigger pendulum makes one full oscillation, the smaller pendulum will make $\left(1 + \frac{1}{4}\right)$ oscillation. The bigger pendulum will be in the mean position and the smaller one will be in the positive extreme position. Thus, phase difference = 90°.

9. (b) $n = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \Rightarrow n \propto \frac{1}{\sqrt{l}} \Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{l_2}{l_1}} = \sqrt{\frac{L_2}{2L_2}}$

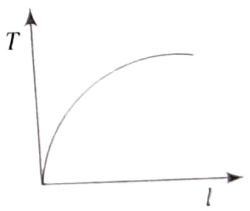
$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{1}{\sqrt{2}} \Rightarrow n_2 = \sqrt{2}\,n_1 \Rightarrow n_2 > n_1$$

Energy, $E = \frac{1}{2}m\omega^2 a^2 = 2\pi^2 m n^2 a^2$

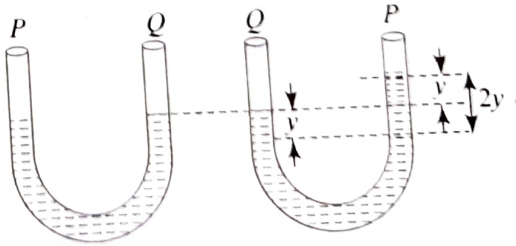
$$\Rightarrow \quad \frac{a_1^2}{a_2^2} = \frac{m_2 n_2^2}{m_1 n_1^2} \quad (\because E \text{ is same})$$

Given $n_2 > n_1$ and $m_1 = m_2 \Rightarrow a_1 > a_2$

10. (b) $T \propto \sqrt{l} \Rightarrow T^2 \propto l$



11. (d) If the level of liquid is depressed by y cm on one side, then the level of liquid in column P is $2y$ cm higher than B as shown.



The weight of extra liquid on the side $P = 2Aydg$.
This becomes the restoring force on mass M.

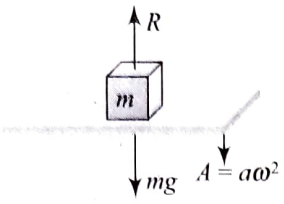
$\therefore$ Restoring acceleration $= \frac{-2Aydg}{M}$

This relation satisfies the condition of SHM, i.e., $a \propto -y$.

Hence, time period, $T = 2\pi\sqrt{\frac{\text{Displacement}}{|\text{Acceleration}|}} = 2\pi\sqrt{\frac{y}{\frac{2Aydg}{M}}}$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{M}{2Adg}}$

12. (a) By drawing free body diagram of object during the downward motion at extreme position, for equilibrium of mass
$mg = R = mA$ (A = Acceleration)
For critical condition $R = 0$
so $mg = mA \Rightarrow mg = ma\omega^2$



$\Rightarrow \quad \omega = \sqrt{g/a} = \sqrt{\frac{9.8}{3.92\times10^{-3}}} = 50$

$\Rightarrow \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{50} = 0.1256\,\text{sec}$

13. (d) When the bob is immersed in water its effective weight $= \left(mg - \frac{m}{\rho}g\right) = mg\left(\frac{\rho-1}{\rho}\right)$

$\therefore g_{\text{eff}} = g\left(\frac{\rho-1}{\rho}\right)$

$\frac{T'}{T} = \sqrt{\frac{g}{g_{\text{eff}}}} \Rightarrow T' = T\sqrt{\frac{\rho}{(\rho-1)}}$

14. (a) As a is the side of cube σ is its density.

Mass of cube is $a^2\sigma$ its weight $= a^3\sigma g$

Let h be the height of cube immersed in liquid of density ρ in equilibrium then, $F = a^2h\rho g = Mg = a^3\sigma g$

If it is pushed down by y then the buoyant force $F' = a^2(h+y)\rho g$

Restoring force is $\Delta F = F' - F = a^2(h+y)\sigma g - a^2h\sigma g$
$= a^2y\rho g$

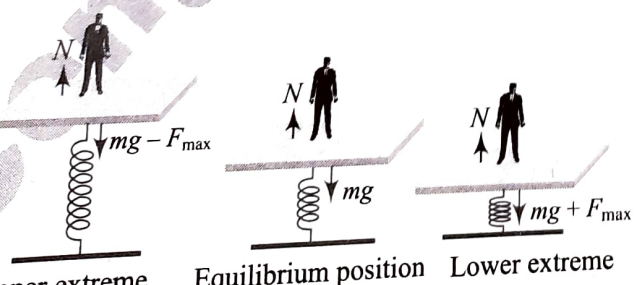
Restoring acceleration $= \frac{\Delta F}{M} = \frac{a^2y\rho g}{M} = -\frac{a^2\rho g}{a^2\sigma}y$

Motion is S.H.M. $\Rightarrow \quad T = 2\pi\sqrt{\frac{a^3\sigma}{a^2\rho g}} = 2\pi\sqrt{\frac{a\sigma}{\rho g}}$

15. (d) The maximum force acting on the body executing simple harmonic motion is

$$m\omega^2 a = m\times(2\pi f)^2 a = 60\times\left(2\pi\times\frac{2}{\pi}\right)^2\times0.1\,\text{N}$$

$= 60\times16\times0.1 = 96\,\text{N} = \frac{96}{9.8} \approx 10\,\text{kgf}$ and this force is towards mean position.



The reaction of the force on the platform away from the mean position. It reduces the weight of man on upper extreme i.e. net weight = (60 – 10) kgf.
This force adds to the weight at lower extreme position i.e. net weight becomes = (60 + 10) kgf.
Therefore, the reading the weight recorded by spring balance fluctuates between 50 kgf and 70 kgf.

DPP 5.5

Subjective Type

1.

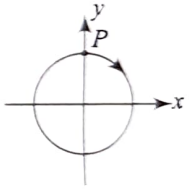
$$T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{m\frac{(a^2+a^2)}{12} + m\left(\frac{a}{\sqrt{2}}\right)^2}{mg\cdot\frac{a}{\sqrt{2}}}}$$

$$= 2\pi\sqrt{\frac{\left(\frac{a}{6}+\frac{a}{2}\right)\cdot\sqrt{2}}{g}} = 2\pi\sqrt{\frac{2\sqrt{2}a}{3g}}$$

2. (a) $x^2 + y^2 = a^2(\sin^2\omega t + \cos^2\omega t) = a^2$

Hence the particle moves in a circle of radius 'a' with centre at origin.

(b) At $t = 0$ sec, $x = 0$ and $y = +a$. Hence the particle is at P as shown in figure.
As 't' increases 'x' increases and 'y' decreases
$\therefore$ The particle moves in clock wise sense.



(c) The 'x' and 'y' components of acceleration are

$$a_x = \frac{d^2x}{dt^2} = -\omega^2x;\ a_y = \frac{d^2y}{dt^2} = -\omega^2y$$

The magnitude of force $F = m\sqrt{a_x^2 + a_y^2} = m\omega^2a$

Single Correct Answer Type

3. (c) $y = 0.2\sin(10\pi t + 1.5\pi)\cos(10\pi t + 1.5\pi)$
$= 0.1\sin 2(10\pi t + 1.5\pi)$ [$\because \sin 2A = 2\sin A\cos A$]
$= 0.1\sin(20\pi t + 3.0\pi)$

$\therefore$ Time period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{20\pi} = \frac{1}{10} = 0.1\,\text{sec}$

4. (b) $x = 12\sin\omega t - 16\sin^3\omega t = 4[3\sin\omega t - 4\sin^3\omega t]$
$= 4[\sin 3\omega t]$ (By using $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$) (By using $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$)

$\therefore$ maximum acceleration $A_{\max} = (3\omega)^2\times4 = 36\omega^2$

5. (c) Given, $x = a\cos\omega t$ (i)

$y = a\sin\omega t$ (ii)

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Squaring and adding Eqs. (i) and (ii),

$x^2 + y^2 = a^2$

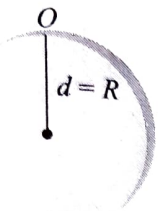
$\Rightarrow \quad x^2 + y^2 = a^2 \qquad (\cos^2 \omega t + \sin^2 \omega t)$

$[\because \cos^2 \omega t + \sin^2 \omega t = 1]$

This is the equation, of a circle

Clearly, the locus is a circle of constant radius a.

6. (d) Time period of a physical pendulum,



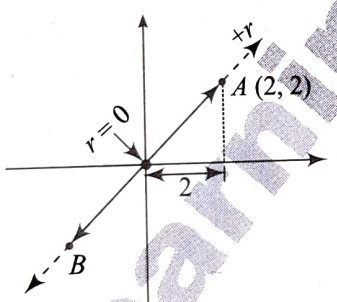
$$T = 2\pi\sqrt{\frac{I_0}{mgd}} = 2\pi\sqrt{\frac{\left(\frac{1}{2}mR^2 + mR^2\right)}{mgR}}$$

$$= 2\pi\sqrt{\frac{3R}{2g}} \qquad \text{(i)}$$

$$T_{\text{simple pendulum}} = 2\pi\sqrt{\frac{l}{g}} \qquad \text{(ii)}$$

Equating (i) and (ii), $l = \frac{3}{2}R$

7. (b) Let the line joining AB represents axis 'r'. By the conditions given 'r' coordinate of the particle at time t is $r = 2\sqrt{2}\cos \omega t$



$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$$

$\therefore \quad r = 2\sqrt{2}\cos \pi t$

$$x = r\cos 45° = \frac{r}{\sqrt{2}} = 2\cos \pi t$$

$\therefore \quad a_s = -\omega^2 x = -\pi^2\, 2\cos \pi t$

$\therefore \quad F_s = ma_x = -4\pi^2 \cos \pi t$

Multiple Correct Answers Type

8. (a, d)

According to the question, the displacement

$y = a \sin \omega t + b \cos \omega t$

Let $a = A \sin \phi$ and $b = A \cos \phi$

Now, $a^2 + b^2 = A^2 \sin^2 \phi + A^2 \cos^2 \phi$

$= A^2 \Rightarrow A = \sqrt{a^2 + b^2}$

$y = A \sin \phi . \sin \omega t + A \cos \phi . \cos \omega t$

$= A \sin (\omega t + \phi)$

$\frac{dy}{dt} = A\omega \cos (\omega t + \phi)$

$\frac{d^2y}{dt^2} = -A\omega^2 \sin(\omega t + \phi)$

$= -Ay\omega^2 = (-A\omega^2)y \quad \Rightarrow \frac{d^2y}{dt^2} \propto (-y)$

Hence, it is an equation of SHM with amplitude

$A = \sqrt{a^2 + b^2}$

9. (a, c)

$y = y_1 + y_2 + y_3$

$= a\sin \omega t + a\sin\left(\omega t + \frac{\pi}{4}\right) + a\sin\left(\omega t + \frac{\pi}{2}\right)$

$= a[\sin \omega t + \sin (\omega t + \pi/2)] + a \sin (\omega t + \pi/4)$

$= 2a \sin (\omega t + \pi/4) \cos \pi/4 + a \sin (\omega t + \pi/4)$

$= (\sqrt{2} + 1)\, a \sin (\omega t + \pi/4)$

So, amplitude, $A = (\sqrt{2} + 1)a$, which differ in phase by $\pi/4$ relative to the first.

Energy $\propto$ (Amplitude)^2

$$\frac{E_{\text{resultant}}}{E_{\text{single}}} = \left(\frac{A}{a}\right)^2 = (\sqrt{2} + 1)^2 = (3 + 2\sqrt{2})$$

10. (b, d)

$x = -B$ and $C = 2B$

$x = B \cos 2\omega t + B \sin 2\omega t$

$= B\sqrt{2} \sin (2\omega t + \pi/4)$

Amplitude of SHM is $B\sqrt{2}$.

$x = B + B \sin 2\omega t$ for $A = B$ and $C = 2B$

It is also equation of simple harmonic motion about point $x = B$. Between $x = 0$ and $x = 2B$ with amplitude B, the function oscillates.

11. (b, d)

$x = 3 \sin 100t + 8 \cos^2 50t$

$= 3 \sin 100t + \frac{8[1 + \cos 100t]}{2}$

$x = 4 + 3 \sin 100t + 4 \cos 100t$

$(x - 4) = 5 \sin (100t + \phi) \qquad \left\{\tan \phi = \frac{4}{3}\right\}$

Amplitude = 5 units

Maximum displacement = 9 units.

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CHAPTER 6

DPP 6.1

Single Correct Answer Type

1. (c) $y = \log\frac{x^2 - t^2}{x - t} = \log(x + t)$

$\left(\log a - \log b = \log\frac{a}{b}\right)$

$\frac{\partial y}{\partial x} = \frac{1}{(x+t)}$

$\frac{\partial^2 y}{\partial x^2} = -\frac{1}{(x+t)^2}$ and $\frac{\partial y}{\partial t} = \frac{(\partial x/\partial t)}{(x+t)} = \frac{v}{(x+t)}$

$\frac{\partial^2 y}{\partial t^2} = -\frac{v^2}{(x+t)^2} \Rightarrow \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$

which is the general form of wave equation.

2. (b) $y(x, t=0) = \frac{6}{x^2}$ then $y(x,t) = \frac{6}{(x-2t)^2}$

$\Rightarrow \quad \frac{\partial y}{\partial t} = \frac{24}{(x-2t)^3}$ at $x = 2, t = 2$

$V_y = \frac{24}{(-2)^3} = -3$ m/s

3. (c) A wave travelling in positive x-direction may be represented as $y = A\sin\frac{2\pi}{\lambda}(vt - x)$. On putting values,

$y = 0.2\sin\frac{2\pi}{60}(360t - x)$

$\Rightarrow \quad y = 0.2\sin 2\pi\left(6t - \frac{x}{60}\right)$

4. (a) The given equation can be x written as

$y = \frac{A}{2}\cos\left(4\pi nt - \frac{4\pi x}{\lambda}\right) + \frac{A}{2} \quad \left(\because \cos^2\theta = \frac{1+\cos 2\theta}{2}\right)$

Hence, amplitude $= \frac{A}{2}$ and frequency $= \frac{\omega}{2\pi} = \frac{4\pi n}{2\pi} = 2n$

and wavelength $= \frac{2\pi}{k} = \frac{2\pi}{4\pi/\lambda} = \frac{\lambda}{2}$.

5. (b) $y_1 = a_1\sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$ and

$y_2 = a_2\cos\left(\omega t - \frac{2\pi x}{\lambda} + \phi\right) = a_2\sin\left(\omega t - \frac{2\pi x}{\lambda} + \phi + \frac{\pi}{2}\right)$

So, phase difference $= \phi + \frac{\pi}{2}$ and $\Delta = \frac{\lambda}{2\pi}\left(\phi + \frac{\pi}{2}\right)$

6. (b) Equation of A, B, C and D are

$y_A = A\sin\omega t$, $y_B = A\sin(\omega t + \pi/2)$

$y_C = A\sin(\omega t - \pi/2)$, $y_D = A\sin(\omega t - \pi)$

It is clear that wave C lags behind by a phase angle of $\pi/2$ and the wave B is ahead by a phase angle at $\pi/2$.

7. (d) Given equation $y = y_0\sin(\omega t - \phi)$

at $t = 0$, $y = -y_0\sin\phi$

this is the case with curve marked D.

8. (d) Points B and F are in same phase as they are λ distance apart.

9. (c) Here $A = 0.05$ m, $\frac{5\lambda}{2} = 0.025 \Rightarrow \lambda = 0.1$ m

Now standard equation of wave

$y = A\sin\frac{2\pi}{\lambda}(vt - x) \Rightarrow y = 0.05\sin 2\pi(3300t - 10x)$

10. (b) At $t = 0$ and $x = \frac{\pi}{2k}$. The displacement

$y = a_0\sin\left(\omega x_0 - k \times \frac{\pi}{2x}\right) = -a_0\sin\frac{\pi}{2} = -a_0$

From graph, point of maximum displacement (a_0) in negative direction is Q.

Multiple Correct Answers Type

11. (a, b, c, d)

On comparing the given equation with $y = a\sin(\omega t + kx)$, it is clear that wave is travelling in negative x-direction.

Its amplitude $a = 10^4$ m and $\omega = 60$, $k = 2$. Hence frequency

$n = \frac{\omega}{2\pi} = \frac{60}{2\pi} = \frac{30}{\pi}$ Hz

$k = \frac{2\pi}{\lambda} = 2 \Rightarrow \lambda = \pi$ m and $v = \frac{\omega}{k} = \frac{60}{2} = 30$ m/s

12. (a, c)

Speed $= n\lambda = n(4ab) = 4n \times ab$ $\left(\text{as } ab = \frac{\lambda}{4}\right)$

Path difference between b and e is $\frac{3\lambda}{4}$.

So the phase difference $= \frac{2\pi}{\lambda}$.

Path difference $= \frac{2\pi}{\lambda}\cdot\frac{3\lambda}{4} = \frac{3\pi}{2}$

Comprehension Type

13. (b) Given $\omega = 3\pi$

$\therefore \quad f = \frac{\omega}{2\pi} = 1.5$

Also $\Delta x = 1.0$ cm

Given, $\phi = \frac{2\pi}{\lambda}\Delta x \quad \Rightarrow \quad \frac{\pi}{8} = \frac{2\pi}{\lambda} \times 1$

$\Rightarrow \quad \lambda = 16$ cm $\Rightarrow$ v = fλ = 16 × 1.5 = 24 cm/sec

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14. (b) $y = a\sin\frac{2\pi}{\lambda}(vt - x)$

$y = 2\sin\frac{2\pi}{\lambda}(24t - x)$

for $t = 1, x = 4$ cm

$$y = 2\sin\left(\frac{2\pi}{16}\times 20\right) = 2\sin\left(2\pi + \frac{\pi}{2}\right)$$

$y = 2$ cm

Fill in the Blanks Type

15. Comparing given equation with standard equation of progressive wave. The velocity of wave

$$v = \frac{\omega(\text{Co-efficient of } t)}{k(\text{Co-efficient of } x)} = \frac{200\pi}{0.5\pi} = 400 \text{ cm/s}$$

16. Comparing with $y = a\cos(\omega t + kx - \phi)$,

We get $k = \frac{2\pi}{\lambda} = 0.02 \Rightarrow \lambda = 100$ cm

Also, it is given that phase difference between particles $\Delta\phi = \frac{\pi}{2}$. Hence path difference between them

$$\Delta = \frac{\lambda}{2\pi}\times\Delta\phi = \frac{\lambda}{2\pi}\times\frac{\pi}{2} = \frac{\lambda}{4} = \frac{100}{4} = 25 \text{ cm}$$

17. Let the phase of second particle be ϕ. Hence phase difference between two particles is $\Delta\phi = \frac{2\pi}{\lambda}\Delta x$

$$\Rightarrow \left(\phi - \frac{\pi}{3}\right) = \frac{2\pi}{60}\times 15 \Rightarrow \phi - \frac{\pi}{3} = \frac{\pi}{2} \Rightarrow \phi = \frac{5\pi}{6}$$

18. The given equation can be written as

$$y = 4\sin\left(4\pi t - \frac{\pi x}{16}\right) \Rightarrow (v) = \frac{\text{Co-efficient of } t(\omega)}{\text{Co-efficient of } x(K)}$$

$$\Rightarrow v = \frac{4\pi}{\pi/16} = 64 \text{ cm/sec along } +x \text{ direction.}$$

19. The given equation representing a wave travelling along $-y$ direction (because '+' sign is given between t term and x term). On comparing it with $x = A\sin(\omega t + ky)$

We get $k = \frac{2\pi}{\lambda} = 12.56 \Rightarrow \lambda = \frac{2\times 3.14}{12.56} = 0.5$ m

20. $y_1 = 10^{-6}\sin[100t + (x/50) + 0.5]$

$$y_2 = 10^{-6}\sin\left[100t + \left(\frac{x}{50}\right) + \left(\frac{\pi}{2}\right)\right]$$

Phase difference ϕ
$= [100t + (x/50) + 1.57] - [100t + (x/50) + 0.5]$
$= 1.07$ rad

DPP 6.2

Subjective Type

1. The instantaneous power through any point of the string is

$p = 4\pi^2 f^2 A^2 \mu V \cos^2(\omega t - Kx)$

At $x = 0$, $p = 4\pi^2 f^2 A^2 \mu V \cos^2 \omega t$

Total energy transferred

$$= \int_{t=0}^{\pi/12\omega} p\,dt = 4\pi^2 f^2 A^2 v\mu \int_0^{\pi/12\omega} \cos^2\omega t\,dt$$

$$= 4\pi^2 f^2 A^2 v\mu \int_0^{\pi/12w} \frac{1+\cos 2\omega t}{2}\,dt$$

$$= 2\pi^2 f^2 A^2 v\mu\left[t + \frac{\sin 2\omega t}{2\omega}\right]_0^{\pi/12\omega}$$

$$= 2\pi^2 f^2 A^2 v\mu\left[\frac{\pi}{12\omega} + \frac{1}{2\omega}\left(\sin 2\omega\cdot\frac{\pi}{12\omega}\right)\right]$$

$$= 2\pi^2 f^2 A^2 v\mu\left[\frac{\pi}{12\omega} + \frac{1}{2\omega}\cdot\frac{1}{2}\right]$$

$$= 2\pi^2\frac{\omega^2}{4\pi^2}A^2\frac{\omega}{K}\mu\left[\frac{\pi+3}{12\omega}\right]$$

$$E = \frac{(\pi+3)\mu\omega^2 A^2}{24K}$$

Single Answer Correct Type

2. (d) SHM equation of the particle at $x = 4$ is

$$y = 4\sin\left[\omega(t+2) + \frac{\pi}{6}\right]$$

$\therefore$ Wave equation replacing t by $\left[t - \left(\frac{x-4}{v}\right)\right]$ is

$$y = 4\sin\left(\omega\left(t - \frac{(x-4)}{v} - 2\right) + \frac{\pi}{6}\right)$$

$$= 4\sin\left[\omega(t-2) - k(x-4) + \frac{\pi}{6}\right]$$

3. (d) Particle velocity $(v_p) = -v \times$ Slope of the graph at that point
At point 1: Slope of the curve is positive, hence particle velocity is negative or downward (↓).
At point 2: Slope negative, hence particle velocity is positive or upwards (↑).
At point 3: Again slope of the curve is positive, hence particle velocity is negative or downward (↓).

4. (c) The particle velocity is maximum at B and is given by

$$\frac{dy}{dt} = (v_p)_{\max} = \omega A$$

Also wave velocity is $\frac{dx}{dt} = v = \frac{\omega}{k}$

So slope $\frac{dy}{dx} = \frac{(v_p)_{\max}}{v} = kA$

5. (a) The given equation is $y = 10\sin(0.01\pi x - 2\pi t)$
Hence ω = coefficient of $t = 2\pi$
$\Rightarrow$ Maximum speed of the particle

$$v_{\max} = a\omega = 10\times 2\pi = 10\times 2\times 3.14 = 62.8 \approx 63 \text{ cm/s}$$

6. (d) On comparing the given equation with standard equation $y = a\sin\frac{2\pi}{\lambda}(vt - x)$. It is clear that wave speed $(v)_{\text{wave}} = v$ and maximum particle velocity $(v_{\max})_{\text{particle}} = a\omega = y_0 \times$ co-efficient of $t = y_0 \times \frac{2\pi v}{\lambda}$

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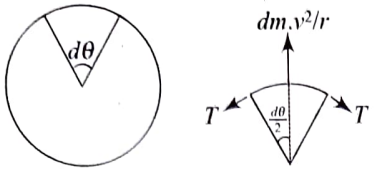
$\because \quad (v_{max})_{particle} = 2(\omega)_{wave} \Rightarrow \dfrac{a \times 2\pi v}{\lambda} = 2v \Rightarrow \lambda = \pi y_0$

7. (b) $dm \cdot \omega^2 R = 2T \sin\dfrac{d\theta}{2}$



$\mu R\, d\theta\, \omega^2 R = 2T \dfrac{d\theta}{2}$

$\Rightarrow \quad \mu\omega^2 R^2 = T$

$\Rightarrow \quad v_W = \sqrt{\dfrac{T}{\mu}} = \sqrt{\omega^2 R^2} = \omega R$

Also speed of string is ωR

The velocity of disturbance w.r.t. ground = $\omega R + \omega R = 2\omega R$

8. (d) $V = \sqrt{\dfrac{T}{\lambda}} = \dfrac{\omega}{k}$

$\Rightarrow \quad T = \dfrac{\omega^2\lambda}{k^2} = \left(\dfrac{420}{21}\right)^2 \times 0.2 = 80\ \text{N}$

9. (b) Equation of wave is $y = A \sin(\omega t - kx)$

$\Rightarrow \quad \dfrac{A}{2} = A \sin\left(\dfrac{2\pi}{T} \times \dfrac{T}{6} - \dfrac{2\pi}{\lambda} \times 4\right)$

$\Rightarrow \quad \dfrac{\pi}{6} = \dfrac{\pi}{3} - \dfrac{2\pi}{\lambda} \times 4 \Rightarrow \dfrac{2\pi}{\lambda} \times 4 = \dfrac{\pi}{6}$

$\lambda = 48$ cm

$\lambda = 0.48$ m

10. (a) $P = \dfrac{1}{2}\mu\omega^2 A^2 V$ and $v = \sqrt{\dfrac{T}{\mu}}$ will not change as both T and μ are constant. ω will also not change as it is property of the source only that is causing the wave motion. Hence, to make power half, the amplitude becomes $\dfrac{A_0}{\sqrt{2}}$.

11. (a) The amplitude A is half of the 6.00 mm vertical range shown in the figure, that is, $A = 3.0$ mm.

The speed of the wave is $v = d/t = 15$ m/s, where $d = 0.060$ m and $t = 0.0040$ s. The angular wave number is $k = 2\pi/\lambda$ where $\lambda = 0.40$ m. Thus, $k = \dfrac{2\pi}{\lambda} = 16$ rad/m.

The angular frequency is found from

$\omega = kv = (16\ \text{rad/m})(15\ \text{m/s}) = 2.4 \times 10^2\ \text{rad/s}$

We choose the minus sign (between kx and ωt) in the argument of the sine function because the wave is shown traveling to the right (in the $+x$ direction). Therefore, with SI units understood, we obtain $y = A \sin[kx - \omega t]$

$= (3\ \text{mm}) \sin[16x - 2.4 \times 10^2 t]$

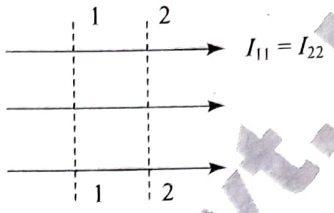
12. (d) Intensity $\propto a^2\omega^2$

Here $\dfrac{a_A}{a_B} = \dfrac{2}{1}$ and $\dfrac{\omega_A}{\omega_B} = \dfrac{1}{2} \Rightarrow \dfrac{I_A}{I_B} = \left(\dfrac{2}{1}\right)^2 \times \left(\dfrac{1}{2}\right)^2 = \dfrac{1}{1}$

Multiple Correct Answers Type

13. (a, c, d)

For a travelling wave, the intensity of wave remains constant if it is a plane wave.



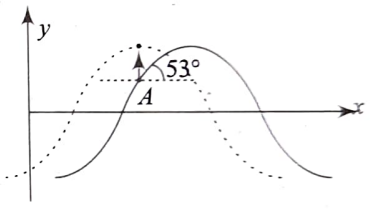
Intensity of wave is inversely proportional to the square of the distance from the source if the wave is spherical, i.e.,

$I = \dfrac{P}{4\pi r^2}$

Intensity of spherical wave on the spherical surface centred at source always remains same. Here total intensity means power P.

14. (b, c)

As shown in the curve, if wave is moving along $-x$ axis, V_P is positive.



$\dfrac{V_p}{V_w} = -\tan\theta$

$|-\tan\theta| > 1 \qquad \therefore \quad V_P > V_W$

15. (a, b, c, d)

The general expression for $y(x, t)$ for the wave is $y(x, t) = y_m \sin(kx - \omega t)$, which, at $x = 10$ cm, becomes $y(x = 10\ \text{cm}, t) = y_m \sin[k(10\ \text{cm} - \omega t)]$.

Comparing this with the expression given, we find $\omega = 4.0$ rad/s, or $f = \omega/2\pi = 0.64$ Hz.

Since $k(10\ \text{cm}) = 1.0$, the wave number is $k = 0.10$/cm. Consequently, the wavelength is $\lambda = 2\pi/k = 63$ cm.

The amplitude is $y_m = 5.0$ cm.

The sign is minus since the wave is traveling in the $+x$ direction.

Summarizing the results obtained above by substituting the values of k and co into the general expression for $y(x, t)$, with centimeters and seconds understood, we obtain,

$y(x, t) = 5.0 \sin(0.10x - 4.0t)$.

Since $v = \dfrac{\omega}{k} = \sqrt{\dfrac{T}{\mu}}$, the tension is

$T = \dfrac{\omega^2\mu}{k^2} = \dfrac{(4.0\ \text{g/cm})(4.0\ \text{s}^{-1})^2}{(0.10\ \text{cm}^{-1})^2}$

$= 6400\ \text{g} \cdot \text{cm/s}^2 = 0.064\ \text{N}$

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CHAPTER 7

DPP 7.1

Single Correct Answer Type

1. (d) $\lambda = \frac{v}{n} = \frac{350}{350} = 1\text{ m} = 100\text{ cm}$

Also path difference (Δx) between the waves at the point of observation is $AP = BP = 25$ cm. Hence

$$\Rightarrow \quad \Delta\phi = \frac{2\pi}{\lambda}(\Delta x) = \frac{2\pi}{1} \times \left(\frac{25}{100}\right) = \frac{\pi}{2}$$

$$\Rightarrow \quad A = \sqrt{(a_1)^2 + (a_2)^2} = \sqrt{(0.3)^2 + (0.4)^2} = 0.5\text{ mm}$$

2. (d) Path difference (Δx) = 50 cm = $\frac{1}{2}$ m

$\therefore$ Phase difference $\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x \Rightarrow \phi = \frac{2\pi}{1} \times \frac{1}{2} = \pi$

Total phase difference = $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\Rightarrow A = \sqrt{a^2 + a^2 + 2a^2\cos(2\pi/3)} = a$

3. (c) If two waves of nearly equal frequency superpose, they give beats if they both travel in straight line and $I_{max} = 0$ if they have equal amplitudes.

4. (c) For interference, two waves must have a constant phase relationship. Equations '1' and '3' and '2' and '4' have a constant phase relationship of $\frac{\pi}{2}$ out of two choices. Only one S_2 emitting '2' and S_4 emitting '4' is given. Hence, only (c) option is correct.

5. (a) The resultant amplitude is given by

$$A_R = \sqrt{A^2 + A^2 + 2AA\cos\theta} = \sqrt{2A^2(1 + \cos\theta)}$$

$$= 2A\cos\theta/2 \qquad (\because H\cos\theta = 2\cos^2\theta/2)$$

6. (d) $y = \frac{1}{\sqrt{a}}\sin\omega t \pm \frac{1}{\sqrt{b}}\sin\left(\omega t + \frac{\pi}{2}\right)$

Here phase difference = $\frac{\pi}{2}$. $\therefore$ The resultant amplitude

$$= \sqrt{\left(\frac{1}{\sqrt{a}}\right)^2 + \left(\frac{1}{\sqrt{b}}\right)^2} = \sqrt{\frac{1}{a} + \frac{1}{b}} = \sqrt{\frac{a+b}{ab}}$$

7. (b) Superposition of waves does not alter the frequency of the resultant wave and resultant amplitude

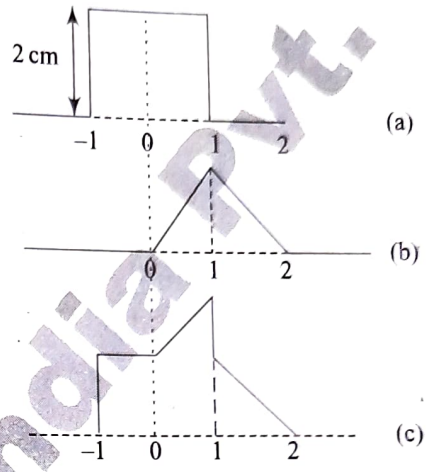
$\Rightarrow a^2 = a^2 + a^2 + 2a^2\cos\phi = 2a^2(1 + \cos\phi)$

$\Rightarrow \cos\phi = -1/2 = \cos 2\pi/3 \therefore \phi = 2\pi/3$

8. (d) As wave has been reflected from a rarer medium, therefore there is no change in the phase. Hence the equation for the opposite direction can be written as

$$y = 0.5A\sin(-kx - \omega t + \theta)$$
$$= -0.5A\sin(kx + \omega t - \theta)$$

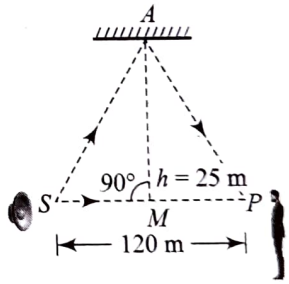
9. (d) At $t = 2$ second, the positions of both pulses are separately given by Fig. (a) and Fig. (b); the superposition of both pulses is given by Fig. (c)



10. (a) Let S be source of sound and P the person or listener.

The waves from S reach point P directly following the path SMP and being reflected from the ceiling at point A following the path SAP. M is mid-point of SP (i.e. $SM = MP$) and $\angle SMA = 90°$

Path difference between waves $\Delta x = SAP - SMP$



We have $SAP = SA + AP = 2(SA)$

$$= 2\sqrt{[(SM)^2 + (MA)^2]} = 2\sqrt{(60^2 + 25^2)} = 130\text{ m}$$

$\therefore$ Path difference = $SAP - SMP = 130 - 120 = 10$ m

Path difference due to reflection from ceiling = $\frac{\lambda}{2}$

$\therefore$ Effective path difference $\Delta x = 10 + \frac{\lambda}{2}$. For constructive interference,

$$\Delta x = 10 + \frac{\lambda}{2} = n\lambda \Rightarrow (2n-1)\frac{\lambda}{2} = 10 (n = 1, 2, 3...)$$

$\therefore$ Wavelength $\lambda = \frac{2 \times 10}{(2n-1)} = \frac{20}{2n-1}$

The possible wavelengths are $\lambda = 20, \frac{20}{3}, \frac{20}{5}, \frac{20}{7}, \frac{20}{9}, \ldots$

$= 20$ m, 6.67 m, 4 m, 2.85 m, 2.22 m, ...

11. (b) Initially $S_1M = S_2M$

$\Rightarrow$ Path difference (Δx) = $S_1M - S_2M = 0$

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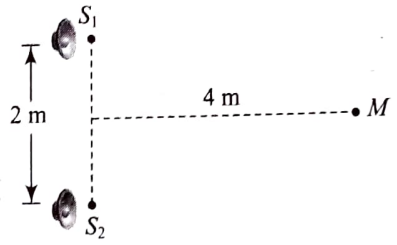
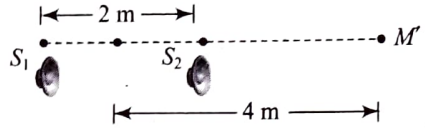


Finally when the box is rotated

Path difference = $S_1M' - S_2M' \Rightarrow \Delta x = 5 - 3 = 2$ m



For maxima,

Path difference = (Even multiple) $\frac{\lambda}{2} \Rightarrow \Delta x = (2n)\frac{\lambda}{2}$

For 5 maximum responses,

$\Rightarrow \quad 2 = 2(5)\frac{\lambda}{2}\left\{\because \Delta x = (2n)\frac{\lambda}{2}\right\} \Rightarrow \lambda = \frac{2}{5} = 0.4$ m

12. (b) Path difference between the wave reaching at D

$\Delta x = L_2P - L_1P = \sqrt{40^2 + 9^2} - 40 = 41 - 40 = 1$ m

For maximum $\Delta x = (2n)\frac{\lambda}{2}$

For first maximum $(n = 1) \Rightarrow 1 = 2(1)\frac{\lambda}{2} \Rightarrow \lambda = 1$ m

$\Rightarrow \quad n = \frac{v}{\lambda} = 330$ Hz

13. (c) After two seconds each wave travel a distance of $2.5 \times 2 = 5$ cm, i.e., the two pulses will meet in mutually opposite phase and hence the amplitude of resultant will be zero.

Multiple Correct Answers Type

14. (a, b, d)

Resultant displacement, $y = y_1 + y_2$

For y to be zero, $y = 0$

$(2x - 3t)^2 = (2x + 3t - 6)^2$

On solving $\left(x - \frac{3}{2}\right)(t - 1) = 0$

Therefore at $x = \frac{3}{2}$, resultant displacement is zero for all values of t.

Fill in the Blanks Type

15. With path difference $\frac{\lambda}{2}$, waves are out of phase at the point of observation.

16. $A_{max} = \sqrt{A^2 + A^2} = A\sqrt{2}$, frequency will remain the same, i.e., ω.

17. Phase difference is 2π means constructive interference so resultant amplitude will be maximum.

18. Resultant amplitude

$$A = \sqrt{a^2 + a^2 + 2aa\cos\phi} = \sqrt{4a^2\cos^2\left(\frac{\phi}{2}\right)}$$

$\because \quad I \propto A^2 \Rightarrow I \propto 4a^2$

19. $A^2 = a^2 = a^2 + a^2 + 2a^2\cos\theta \Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = \frac{2\pi}{3}$

20. Resultant amplitude = $\sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$

$$= \sqrt{0.3^2 + 0.4^2 + 2 \times 0.3 \times 0.4 \times \cos\frac{\pi}{2}} = 0.5 \text{ cm}$$

21. In the same phase $\phi = 0$, so resultant amplitude = $a_1 + a_2$ $= 2A + A = 3A$

22. $\frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2 = \frac{1}{16} \Rightarrow \frac{a_1}{a_2} = \frac{1}{4}$

23. When the piston is moved through a distance of 8.75 cm, the path difference produced is 2×8.75 cm = 17.5 cm. This must be equal to $\frac{\lambda}{2}$ for maximum to change to minimum.

$\therefore \quad \frac{\lambda}{2} = 17.5$ cm $\Rightarrow \lambda = 35$ cm $= 0.35$ m

So, $v = n\lambda \Rightarrow n = \frac{v}{\lambda} = \frac{350}{0.35} = 1000$ Hz

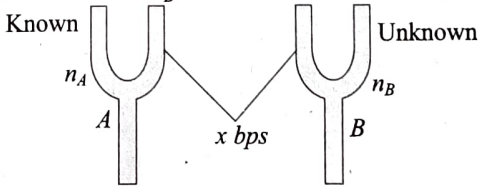
24. $\frac{I_{max}}{I_{min}} = \left(\frac{\frac{a_1}{a_2} + 1}{\frac{a_1}{a_2} - 1}\right)^2 = \left(\frac{\frac{4}{3} + 1}{\frac{4}{3} - 1}\right)^2 = \frac{49}{1}$

DPP 7.2

Single Correct Answer Type

1. (a) Since the source of frequency x gives 8 beats per second with frequency 250 Hz, its possible frequencies are 258 or 242. As the source of frequency x gives 12 beats per second with a frequency 270 Hz, its possible frequency is 282 or 258 Hz. The only possible frequency of x gives 8 beats with frequency 250 Hz and 12 beats per second with 258 Hz as well.

2. (c) Suppose two tuning forks are named A and B with frequencies n_A $= 256$ Hz (known), $n_B = ?$ (unknown), and beat frequency $x = 4$ bps.



Frequency of unknown tuning fork may be

$n_B = 256 + 4 = 260$ Hz

or $\quad = 256 - 4 = 252$ Hz

It is given that on sounding waxed fork A (fork of frequency 256 Hz) and fork B, number of beats (beat frequency) increases. It means that with the decrease in frequency of A, the difference in new frequency of A and the frequency of B has increased. This is possible only when the frequency of A while decreasing is moving away from the frequency of B.

This is possible only if $n_B = 260$ Hz.

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3. (b) n_A = Known frequency = 256 Hz, n_B = ?

$x = 4$ bps, which is decreasing after loading (i.e., $x\downarrow$) also known tuning fork is loaded so $n_A\downarrow$

Hence $n_A\downarrow - n_B = x\downarrow$ (i) Correct

$n_B - n_A\downarrow = x\downarrow$ (ii) Wrong

$\Rightarrow n_B = n_A - x = 256 - 4 = 252$ Hz

4. (c) Time interval between two consecutive beats

$$T = \frac{1}{n_1 - n_2} = \frac{1}{260 - 256} = \frac{1}{4}\text{sec, so } t = \frac{1}{16} = \frac{T}{4}\text{sec}$$

By using the time difference = $\frac{T}{2\pi} \times$ Phase difference

$$\Rightarrow \frac{T}{4} = \frac{T}{2\pi} \times \phi \Rightarrow \phi = \frac{\pi}{2}$$

5. (d) n_A = Known frequency = 341 Hz, n_B = ?

$x = 6$ bps, which is decreasing (i.e., $x\downarrow$) after loading (from 6 to 1 bps)

Unknown tuning fork is loaded so $n_B\downarrow$

Hence $n_A - n_B\downarrow = x\downarrow$ (i) Wrong

$n_B\downarrow - n_A = x\downarrow$ (ii) Correct

$\Rightarrow n_B = n_A + x = 341 + 6 = 347$ Hz

6. (b) n_A = Known frequency = 256 Hz, n_B = ?

$x = 6$ bps, which remains the same after loading.

Unknown tuning fork F_2 is loaded so $n_B\downarrow$

Hence $n_A - n_B\downarrow = x$ (i) Wrong

$n_B\downarrow - n_A = x$ (ii) Correct

$\Rightarrow n_B = n_A + x = 256 + 6 = 262$ Hz

7. (c) Number of beats per second = $n_1 \sim n_2$

$\omega_1 = 2000\pi = 2\pi n_1 \Rightarrow n_1 = 1000$

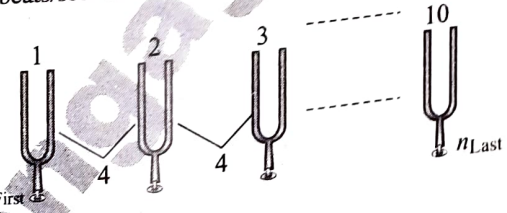
and $\omega_2 = 2008\pi = 2\pi n_2 \Rightarrow n_2 = 1004$

Number of beats heard per sec = $1004 - 1000 = 4$

8. (c) The tuning fork whose frequency is being tested produces 2 beats with oscillator at 514 Hz, therefore, frequency of tuning fork may either be 512 or 516. With oscillator frequency 510 it gives 6 beats/sec, therefore frequency of tuning fork may be either 516 or 504.

Therefore, the actual frequency is 516 Hz, which gives 2 beats/sec with 514 Hz and 6 beats/sec with 510 Hz.

9. (d)



Using $n_{Last} = n_{First} + (N - 1)x$

where N = Number of tuning fork in series

x = beat frequency between two successive forks

$\Rightarrow 2n = n + (10 - 1) \times 4 \Rightarrow n = 36$ Hz

$\therefore n_{First} = 36$ Hz and $n_{Last} = 2 \times n_{First} = 72$ Hz

10. (b) Velocity of sound increases if the temperature increases. So with $v = n\lambda$, if v increases n will increase

at 27°C, $v_1 = n\lambda$, at 31°C, $v_2 = (n + x)\lambda$

Now using $v \propto \sqrt{T}$ $\left(\because v = \sqrt{\frac{\gamma RT}{M}}\right)$

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \frac{n + x}{n}$$

$$\Rightarrow \frac{300 + x}{300} = \sqrt{\frac{(273 + 31)}{(273 + 27)}} = \sqrt{\frac{304}{300}} = \sqrt{\frac{300 + 4}{300}}$$

$$\Rightarrow 1 + \frac{x}{300} = \left(1 + \frac{4}{300}\right)^{1/2} = \left(1 + \frac{1}{2} \times \frac{4}{300}\right) \Rightarrow x = 2$$

$[\because (1 + x)^n = 1 + nx]$

11. (c) Let n be the frequency of fork C then

$$n_A = n + \frac{3n}{100} = \frac{103n}{100} \text{ and } n_B = n - \frac{2n}{100} = \frac{98}{100}$$

but $n_A - n_B = 5 \Rightarrow \frac{5n}{100} = 5 \Rightarrow n = 100$ Hz

$$\therefore \quad n_A = \frac{(103)(100)}{100} = 103 \text{ Hz}$$

12. (c) Beat period $T = \frac{1}{n_1 \sim n_2} = \frac{1}{384 - 380} = \frac{1}{4}$sec. Hence minimum time interval between maxima and minima $t = \frac{T}{2} = \frac{1}{8}$sec.

13. (c) It is given that

n_A = Unknown frequency = ?

n_B = Known frequency = 256 Hz

$x = 3$ bps, which remains the same after loading

Unknown tuning fork A is loaded so $n_A\downarrow$

Hence $n_A\downarrow - n_B = x$ (i) Correct

$n_B - n_A\downarrow = x$ (ii) Wrong

$\Rightarrow n_A = n_B + x = 256 + 3 = 259$ Hz

DPP 7.3

Subjective Type

1. Equation of standing wave can be written as

$y = 2 \sin(kx + \theta) \cos \omega t$

Because the particles at $t = 0$ are at extreme position.

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{4} = \frac{\pi}{2}$$

From the graph it is clear that $x = \frac{1}{2}$, amplitude = 0

$$\therefore \quad 2 \sin\left[\frac{\pi}{2} \cdot \frac{1}{2} + \theta\right] = 0 \Rightarrow \frac{\pi}{4} + \theta = 0 \text{ or } \pi$$

We will select $\frac{\pi}{4} + \theta = \pi$ to suit the initial condition

$$\therefore \theta = \frac{3\pi}{4}$$

$$\therefore \quad y = 2 \sin\left(\frac{\pi}{2}x + \frac{3\pi}{4}\right)\cos\frac{5\pi}{2}t$$

$$= \sin\left(\frac{\pi}{2}x + \frac{3\pi}{4} + \frac{5\pi}{2}t\right) + \sin\left(\frac{\pi}{2}x + \frac{3\pi}{4} - \frac{5\pi}{2}t\right)$$

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Single Correct Answer Type

2. (b) In stationary wave all the particles in one particular segment (i.e., between two nodes) vibrates in the same phase.

3. (a) If $y_{incident} = a \sin(\omega t - kx)$ and $y_{stationary} = a \sin(\omega t)\cos kx$, then it is clear that frequency of both is the same (ω)

4. (d) Particles have kinetic energy maximum at mean position.

5. (a) Waves A and B satisfied the conditions required for a standing wave.

6. (a) Comparing given equation with standard equation

$$y = 2a\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi vt}{\lambda} \text{ gives us } \frac{2\pi}{\lambda} = \frac{\pi}{15} \Rightarrow \lambda = 30$$

Distance between the nearest node and antinodes $= \frac{\lambda}{4} = \frac{30}{4} = 7.5$

7. (b) Since the point $x = 0$ is a node and reflection is taking place from point $x = 0$. This means that reflection must be taking place from the fixed end and hence the reflected ray must suffer an additional phase change of π or a path change of $\frac{\lambda}{2}$.

So, if $y_{incident} = a\cos(kx - \omega t)$

$\Rightarrow \quad y_{reflected} = a\cos(-kx - \omega t + \pi) = -a\cos(\omega t + kx)$

8. (b) Distance between the consecutive node $= \frac{\lambda}{2}$,

but $\lambda = \frac{v}{n} = \frac{20}{n}$ so $\frac{\lambda}{2} = \frac{10}{n}$

9. (d) Minimum time interval between two instants when the string is flat $= \frac{T}{2} = 0.5 \text{ sec} \Rightarrow T = 1 \text{ sec}$

Hence $\lambda = v \times T = 10 \times 1 = 10$ m

10. (a) Waves $z_1 = A\sin(kx - \omega t)$ is travelling towards positive x-direction.

Wave $z_2 = A\sin(kx + \omega t)$ is travelling towards negative x-direction.

Wave $z_3 = A\sin(kx - \omega t)$ is travelling towards positive y, direction.

Since waves z_1 and z_2 are travelling along the same line so they will produce stationary wave.

11. (b) Spacing between successive nodes $= \frac{\lambda}{2}$

Using $V = n\lambda$

$$\lambda \propto V \propto \sqrt{\frac{T}{\mu}}$$

New $\frac{\lambda'}{\lambda} = \frac{\sqrt{2T}}{\sqrt{T}} \Rightarrow \lambda' = \sqrt{2}\lambda$

$$\frac{\lambda'}{2} = \sqrt{2}\cdot\frac{\lambda}{2} = \sqrt{2}x$$

12. (d) At $x = 0$ the phase difference should be π.

$\therefore$ The correct option is (d).

Alternate solution

$y^2 = a\cos(\omega t + kx + \phi_0)$

$$\therefore \quad y = y_1 + y_2 = a\cos\left(\omega t - kx + \frac{\pi}{3}\right) + a\cos(\omega t + kx + \phi_0)$$

$$= 2a\cos\left[\omega t + \frac{\frac{\pi}{3} + \varphi_0}{2}\right] \times \cos\left[kx + \frac{\varphi_0 - \frac{\pi}{3}}{2}\right]$$

$$\therefore \quad y = 0 \text{ at } x = 0 \text{ for any } t \Rightarrow kx + \frac{\varphi_0 - \frac{\pi}{3}}{2} = \frac{\pi}{2} \text{ at } x \neq 0$$

$$\therefore \quad \phi_0 = \frac{4\pi}{3}. \text{ Hence } y_2 = a\cos\left(\omega t + kx + \frac{4\pi}{3}\right)$$

Multiple Correct Answers Type

13. (a,c)

$$y = 2A\sin kx \cdot \sin\omega t$$

$$V_y = \frac{dy}{dt} = 2A\sin kx \cdot \cos\omega t$$

$$V_y = 0 \Rightarrow t = T/4,\ 3T/4 \qquad \left(T = \frac{2\pi}{\omega}\right)$$

(2 times in one time period)

14. (a, c, d)

$$\frac{\lambda}{4} = 0.1 \Rightarrow \lambda = 0.4 \text{ m}$$

From the graph $\Rightarrow T = 0.2$ sec, and amplitude of standing wave is $2A = 4$ cm.

Equation of the standing wave,

$$y(x,t) = -2A\cos\left(\frac{2\pi}{0.4}x\right)\cdot\sin\left(\frac{2\pi}{0.2}t\right) \text{cm}$$

$$y(x = 0.05,\ t = 0.05) = -2\sqrt{2} \text{ cm}$$

$$y(x = 0.04,\ t = 0.025) = -2\sqrt{2}\cos 36°$$

$$\text{Speed} = \frac{\lambda}{T} = 2 \text{ m/sec}$$

$$V_y = \frac{dy}{dt} = -2A \times \frac{2\pi}{0.2}\cos\left(\frac{2\pi x}{0.4}\right)\cdot\cos\left(\frac{2\pi t}{0.2}\right)$$

$$V_y = \left(x = \frac{1}{15}\text{m}, t = 0.1\right) = 20\pi \text{ cm/sec}$$

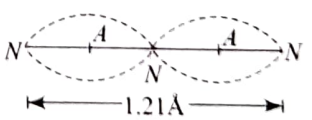
Fill in the Blanks Type

15. On comparing the given equation with standard equation

$$y = 2a\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi vt}{\lambda} \Rightarrow \frac{2\pi x}{\lambda} = \frac{\pi x}{3} \Rightarrow \lambda = 6$$

Separation between two adjacent nodes $= \frac{\lambda}{2} = 3$ cm

16. $\lambda = 1.21$ Å



17. $\frac{\lambda}{4} = 20 \Rightarrow \lambda = 80$ cm, also $\Delta\phi = \frac{\lambda}{2\pi}\cdot\Delta x$

$$\Rightarrow \quad \Delta\phi = \frac{60}{80} \times 2\pi = \frac{3\pi}{2}$$

18. By comparing given equation with $y = a\sin(\omega t)\cos kx$

$$\Rightarrow \quad v = \frac{\omega}{k} = \frac{100}{0.01} = 10^4 \text{ m/s}$$

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19. At fixed end node is formed and distance between two consecutive nodes $\frac{\lambda}{2} = 10$ cm $\Rightarrow \lambda = 20$ cm

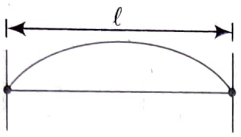
$\Rightarrow u = n\lambda = 20$ m/sec

DPP 7.4

Subjective Type

1. $\mu = \frac{3.2 \text{ gm}}{40 \text{ cm}} = \frac{3.2 \times 10^{-3}}{40 \times 10^{-2}} = \frac{3.2}{40} = \frac{32}{4000}$ kg/m

$l = \frac{\lambda}{2} \Rightarrow \lambda = 2l$ (1)



$f = \frac{v}{\lambda} = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

$\Rightarrow \frac{1000}{64} = \frac{1}{2 \times 40 \times 10^{-2}}\sqrt{\frac{T}{32/4000}}$

$\Rightarrow \left[\frac{1000}{64} \times 2 \times 40 \times 10^{-2}\right]^2 \frac{32}{4000} = T$

$\frac{1000}{64} \times \frac{32}{4000} = T \Rightarrow T = \frac{10}{8}$ N

$y = \frac{10/8}{\frac{10^{-6}}{\frac{.05 \times 10^{-2}}{40 \times 10^{-2}}}} = \frac{10^7}{8}\frac{40}{(.05)} = 10^9$ N/m^2

Single Correct Answer Type

2. (c) Let the frequency of tuning fork be N

As the frequency of vibration string $\propto \frac{1}{\text{length of string}}$

For sonometer wire of length 20 cm, frequency must be $(N+5)$ and that for the sonometer wire of length 21 cm, the frequency must be $(N-5)$ as in each case the tuning fork produces 5 beats/sec with sonometer wire

Hence $n_1 I_1 = n_2 I_2 \Rightarrow (N+5) \times 20 = (N-5) \times 21$

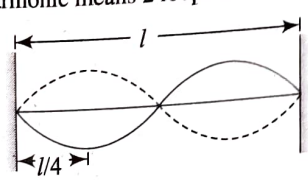
$\Rightarrow N = 205$ Hz

3. (d) $n = \frac{1}{2l}\sqrt{\frac{T}{m}} \Rightarrow n \propto \frac{\sqrt{T}}{l}$

$\Rightarrow \frac{T_2}{T_1} = \left(\frac{n_2}{n_1}\right)^2\left(\frac{l_2}{l_1}\right)^2 = (2)^2\left(\frac{3}{4}\right)^2 = \frac{9}{4}$

4. (c) $v = \sqrt{\frac{T}{m}} \Rightarrow v = \sqrt{\frac{60.5}{(0.035/7)}} = 110$ m/s

5. (a) Second harmonic means 2 loops in a total length



Hence plucking distance from one end $= \frac{l}{2p} = \frac{l}{2 \times 2} = \frac{l}{4}$

6. (d) Frequency in a stretched string is given by

$n = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2 \rho}} = \frac{1}{l}\sqrt{\frac{T}{\pi d^2 \rho}}$ (d = Diameter of the string)

$\Rightarrow \frac{n_1}{n_2} = \frac{l_2}{l_1}\sqrt{\frac{T_1}{T_2} \times \left(\frac{d_2}{d_1}\right)^2 \times \left(\frac{\rho_2}{\rho_1}\right)}$

$= \frac{35}{36}\sqrt{\frac{8}{1} \times \left(\frac{1}{4}\right)^2 \times \frac{2}{1}} = \frac{35}{36} \Rightarrow n_2 = \frac{36}{35} \times 360 = 370$

Hence beat frequency $= n_2 - n_1 = 10$

7. (b) Given equation of stationary wave is

$y = \sin 2\pi \times \cos 2\pi t$, comparing it with standard equation

$y = 2A \sin\frac{2\pi x}{\lambda}\cos\frac{2\pi x}{\lambda}$

We have $\frac{2\pi x}{\lambda} = 2\pi x \Rightarrow \lambda = 1$ m

Minimum distance of string (first mode) $L_{\min} = \frac{\lambda}{2} = \frac{1}{2}$ m

8. (d) $n = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2 \rho}} \Rightarrow n \propto \frac{\sqrt{T}}{lr} \Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{T_1}{T_2}} \times \frac{l_2}{l_1} \times \frac{r_2}{r_1}$

$= \sqrt{\frac{T}{3T}} \times \frac{3l}{l} \times \frac{2r}{r} = 3\sqrt{3} \Rightarrow n_2 = \frac{n}{3\sqrt{3}}$

9. (c) For string $\lambda = \frac{2l}{p}$

where p = number of loops = Order of vibration

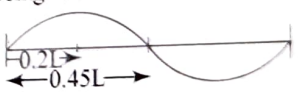
Hence for forth mode $p = 4 \Rightarrow \lambda = \frac{l}{2}$

Hence $v = n\lambda = 500 \times \frac{2}{2} = 500$ m/s

Comprehension Type

10. (b) It is obvious that particle at 0.2 L will have larger amplitude than particle at 0.45 L, 0.5 L being the node and 0.25 being antinode.

11. (a) $k = m\omega^2$ i.e. same for both



12. (d), 13. (b), 14. (b)

Medium of string BC is denser. These will be π phase difference between reflected and transmitted wave.

$\frac{\lambda_r}{\lambda_t} = \frac{v_1}{v_2} = \sqrt{\frac{\mu_2}{\mu_1}}$

Now $\frac{\mu_1}{\mu_2} = \frac{4}{9}$

$\frac{A_r}{A_t} = \frac{\frac{v_2 - v_1}{v_1 + v_2}Ai}{\frac{2v_2}{v_1 + v_2} \cdot Ai} = \frac{v_2 - v_1}{2v_2} = \frac{1}{4}$

Fill in the Blanks Type

15. $n \propto \frac{1}{l} \Rightarrow \frac{n_2}{n_1} = \frac{l_1}{l_2} \Rightarrow n_2 = \frac{l_1}{l_2}n_1 = \frac{1 \times 256}{1/4} = 1024$ Hz

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16. String vibrates in five segments so $\frac{5}{2}\lambda = l \Rightarrow \lambda = \frac{2l}{5}$

Hence $n = \frac{v}{\lambda} = 5 \times \frac{v}{2l} = 5 \times \frac{20}{2 \times 10} = 5$ Hz

17. Here $\frac{\lambda}{2} = 5.0$ cm $\Rightarrow \lambda = 10$ cm

Hence $n = \frac{v}{\lambda} = \frac{200}{10} = 20$ Hz

18. As we know plucking distance from one end $= \frac{l}{2p}$

$\Rightarrow \quad 25 = \frac{100}{2p} \Rightarrow p = 2$. Hence the frequency of vibration

$$n = \frac{p}{2l}\sqrt{\frac{T}{m}} = \frac{2}{2 \times 1}\sqrt{\frac{20}{5 \times 10^{-4}}} = 200 \text{ Hz}$$

19. To produce 5 beats/sec, frequency of one wire should be increased up to 505 Hz, i.e., increment of 1% in basic frequency.

$$n \propto \sqrt{T} \text{ or } T \propto n^2 \Rightarrow \frac{\Delta T}{T} = 2\frac{\Delta n}{n}$$

$\Rightarrow$ Percentage change in tension = 2(1%) = 2%

20. $y = 0.021 \sin(x + 30t) \Rightarrow v = \frac{\omega}{k} = \frac{30}{1} = 30$ m/s

Using, $v = \sqrt{\frac{T}{m}} \Rightarrow 30 = \sqrt{\frac{T}{1.3 \times 10^{-4}}} \Rightarrow T = 0.117$ N

21 $n \propto \sqrt{T}$

$\Rightarrow \quad n_1 : n_2 : n_3 : n_4 = \sqrt{1} : \sqrt{4} : \sqrt{9} : \sqrt{16} = 1 : 2 : 3 : 4$

22. $n = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2 \rho}} \Rightarrow n \propto \frac{\sqrt{T}}{r}$

$\Rightarrow \quad \frac{n_2}{n_1} = \frac{r_1}{r_2}\sqrt{\frac{T_2}{T_1}} = \frac{1}{2} \times \sqrt{\frac{1}{2}} = \frac{1}{2\sqrt{2}}$

23. First overtone of string A = Second overtone of string B.

$\Rightarrow$ Second harmonic of A = Third harmonic of B

$$\Rightarrow \quad n_2 = n_3 \Rightarrow [2(n_1)]_A = [3(n_1)]_B \quad \left(\because \ n_1 = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2 \rho}}\right)$$

$$\Rightarrow \quad 2\left[\frac{1}{2l_A r_A}\sqrt{\frac{T}{\pi\rho}}\right] = 3\left[\frac{1}{2l_B r_B}\sqrt{\frac{T}{\pi\rho}}\right]$$

$$\frac{l_A}{l_B} = \frac{2}{3}\frac{r_B}{r_A} \Rightarrow \frac{l_A}{l_B} = \frac{2}{3} \times \frac{r_B}{(2r_B)} = \frac{1}{3}$$

24. String will vibrate in 7 loops so it will have 8 nodes and 7 antinodes.

Number of harmonics = Number of loops = Number of antinodes

$\Rightarrow$ Number of antinodes = 7

Hence the number of nodes = Number of antinodes + 1

= 7 + 1 = 8

DPP 7.5

Single Correct Answer Type

1. (c) $\lambda_1 = 2l, \lambda_2 = 2l + 2\Delta l \Rightarrow n_1 = \frac{v}{2l}$ and $n_2 = \frac{v}{2l + 2\Delta l}$

$$\Rightarrow \text{ No. of beats } = n_1 - n_2 = \frac{v}{2}\left(\frac{1}{l} - \frac{1}{l + \Delta l}\right) = \frac{v\Delta l}{2l^2}$$

2. (b) For first pipe $n_1 = \frac{v}{4l_1}$ and for second pipe $n_2 = \frac{v}{4l_2}$

So, number of beats $= n_2 - n_1 = 4$

$$\Rightarrow \quad 4 = \frac{v}{4}\left(\frac{1}{l_2} - \frac{1}{l_1}\right) \Rightarrow 16 = 300\left(\frac{1}{l_2} - \frac{1}{1}\right) \Rightarrow l_2 = 94.9 \text{ cm}$$

3. (a) Let l_1 and l_2 be the lengths of closed and open pipes respectively. (Neglecting end correction)

$$l_1 = \frac{\lambda_1}{4} \Rightarrow \lambda_1 = 4l_1 \text{ and } l_2 = \frac{\lambda_2}{2} \Rightarrow \lambda_2 = 2l_2$$

Given $n_1 = n_2$ so $\frac{v}{\lambda_1} = \frac{v}{\lambda_2} \Rightarrow \frac{v}{4l_1} = \frac{v}{2l_2} = \frac{l_1}{l_2} = \frac{1}{2}$

4. (b) Distance between six successive nodes

$$= \frac{5\lambda}{2} = 85 \text{ cm} \Rightarrow \lambda = \frac{2 \times 85}{5} = 34 \text{ cm} = 0.34 \text{ m}$$

Therefore speed of sound in gas

$\Rightarrow \quad n\lambda = 1000 \times 0.34 = 340$ m/s

5. (b) Let the base frequency be n for closed pipe, then notes are n, $3n$, $5n$ .....

$\therefore$ Note $3n = 255 \Rightarrow n = 85$, note $5n = 85 \times 5 = 425$

Note $7n = 7 \times 85 = 595$

6. (c) Number of beats per second,

$$n = \frac{16}{20} = \frac{4}{5} \Rightarrow n = n_1 - n_2 = \frac{v}{4}\left(\frac{1}{l_1} - \frac{1}{l_2}\right)$$

$$\Rightarrow \quad \frac{4}{5} = \frac{v}{4}\left(\frac{1}{1} - \frac{1}{1.01}\right) = \frac{0.01v}{4 \times 1.01}$$

$$v = \frac{16 \times 101}{5} = 323.2 \text{ ms}^{-1}$$

7. (a) For end correction x, $\frac{l_2 + x}{l_1 + x} = \frac{3\lambda/4}{\lambda/4} = 3$

$$x = \frac{l_2 - 3l_1}{2} = \frac{70.2 - 3 \times 22.7}{2} = 1.05 \text{ cm}$$

8. (d) Because the tuning fork is in resonance with air column in the pipe closed at one end, the frequency is $n = \frac{(2N-1)v}{4l}$ where $N = 1, 2, 3$ .... corresponds to different mode of vibration putting $n = 340$ Hz, $v = 340$ m/s, the length of air column in the pipe can be

$$l = \frac{(2N-1)340}{4 \times 340} = \frac{(2N-1)}{4}m = \frac{(2N-1) \times 100}{4}\text{cm}$$

For $N = 1, 2, 3, \ldots$ we get $l = 25$ cm, 75 cm, 125 cm ...

As the tube is only 120 cm long, the length of air column after water is poured in it may be 25 cm or 75 cm only, 125 cm is not possible, the corresponding length of water column in the tube will be (120 – 25) cm = 95 cm or (120 – 75) cm = 45 cm.

Thus minimum length of water column is 45 cm.

9. (c) Critical hearing frequency for a person is 20,000 Hz.

If a closed pipe vibration in N^{th} mode, then frequency of vibration $n = \frac{(2N-1)v}{4l} = (2N-1)n_1$

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(where n_1 = fundamental frequency of vibration)

Hence $20{,}000 = (2N-1)\times 1500 \Rightarrow N = 7.1 \approx 7$

Also, in closed pipe

Number of over tones = (No. of mode of vibration) -1

$= 7 - 1 = 6$

10. (c) Fundamental frequency of closed pipe

$$n = \frac{v}{4l} = 220 \text{ Hz} \Rightarrow v = 220 \times 4l$$

If $\frac{1}{4}$ of the pipe is filled with water, then remaining length of air column is $\frac{3l}{4}$

Now fundamental frequency $= \dfrac{v}{4\left(\frac{3l}{4}\right)} = \dfrac{v}{3l}$ and

First overtone $= 3 \times$ fundamental frequency

$$= \frac{3v}{3l} = \frac{v}{l} = \frac{220 \times 4l}{l} = 880 \text{ Hz}$$

11. (b) Suppose N resonance occurred before tube coming out.

Hence by using $l = \dfrac{(2N-1)v}{4n}$

$$\Rightarrow \quad 1.5 = \frac{(2N-1)\times 330}{4\times 660} \Rightarrow N \approx 6$$

Multiple Correct Answers Type

12. **(a, c, d)**

Fundamental frequency of closed pipe $n = \dfrac{v}{4l}$

where $v = \sqrt{\dfrac{\gamma RT}{M}} \Rightarrow v \propto \dfrac{1}{\sqrt{M}}$

$\because \quad M_{H_2} < M_{air} \Rightarrow v_{H_2} > v_{air}$

Hence fundamental frequency with H_2 will be more as compared to air. So option (a) is correct.

Also $n \propto \dfrac{1}{l}$, hence if l decreases n increases so option (c) is correct.

It is well known that $(n)_{Open} = 2(n)_{Closed}$, hence option (d) is correct.

13. **(a, c)**

$f = n\left(\dfrac{v}{4L}\right)$ for closed organ pipe

$$L = \frac{nv}{4f}, \; n = 1, 3, 5.....$$

$$L_1 = \frac{(1)(330)}{4\times 264} = 31.25 \text{ cm}$$

$$L_2 = 3L_1 = 93.75 \text{ cm}$$

Similarly $L_5 = 5L_1$

Fill in the Blanks Type

14. Fundamental frequency of open pipe is double that of the closed pipe.

15. It is given that

First over tone of closed pipe = First over tone of open pipe

$$\Rightarrow 3\left(\frac{v}{4l_1}\right) = 2\left(\frac{v}{2l_2}\right), \text{ where } l_1 \text{ and } l_2 \text{ are the lengths of closed}$$

and open organ pipes hence $\dfrac{l_1}{l_2} = \dfrac{3}{4}$

16. Minimum audible frequency = 20 Hz.

$$\Rightarrow \quad \frac{v}{4l} = 20 \Rightarrow l = \frac{336}{4\times 20} = 4.2 \text{ m}$$

17. For closed pipe $n_1 = \dfrac{v}{4l} \Rightarrow l = \dfrac{v}{4n} = \dfrac{332}{4\times 166} = 0.5 \text{ m}$

18. $n_{open} = \dfrac{v}{2l_{open}}$

$$n_{closed} = \frac{v}{4l_{closed}} = \frac{v}{4l_{open}/2} = \frac{v}{2l_{open}}$$

$\left(\text{As } l_{closed} = \dfrac{l_{open}}{2}\right)$, i.e., frequency remains unchanged.

19. $n_1 - n_2 = 10$ (i)

Using $n_1 = \dfrac{v}{4l_1}$ and $n_2 = \dfrac{v}{4l_2}$

$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{l_2}{l_1} = \frac{26}{25} \qquad \text{(ii)}$$

After solving these equations $n_1 = 260$ Hz, $n_2 = 250$ Hz

20. Distance between two consecutive nodes

$$= \frac{\lambda}{2} = 46 - 16 = 30 \Rightarrow \lambda = 60 \text{ cm} = 0.6 \text{ m}$$

$\therefore \quad v = n\lambda = 500 \times 0.6 = 300$ m/s

21. For closed pipe

$$l_1 = \frac{v}{4n}, l_2 = \frac{3v}{4n} \Rightarrow v = 2n(l_2 - l_1)$$

$$\Rightarrow \quad n = \frac{v}{2(l_2 - l_1)} = \frac{330}{2\times(0.49 - 0.16)} = 500 \text{ Hz}$$

22. In open organ pipe 5th overtone corresponds to 4th harmonic mode.

Also in open pipe, number of nodes = order of mode of vibration and number of antinodes = (Number of nodes + 1). Here number of nodes = 4, number of antinodes = 4 + 1 = 5.

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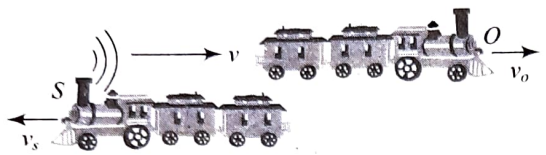
CHAPTER 8

DPP 8.1

Single Correct Answer Type

1. (b)

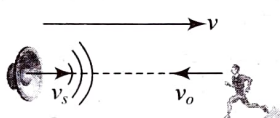


$$n' = n\left(\frac{v - v_o}{v + v_s}\right) = 750\left(\frac{330 - 180 \times \frac{5}{18}}{330 + 108 \times \frac{5}{18}}\right) = 625 \text{ Hz}$$

2. (a) By using $n' = n\left(\frac{v}{v - v_S}\right)$

$$2n = n\left(\frac{v - v_O}{v - 0}\right) \Rightarrow v_0 = -v = -(\text{Speed of sound})$$

Negative sign indicates that observer is moving opposite to the direction of velocity of sound, as shown in the following figure



3. (a) When source is approaching the observer, the frequency heard

$$n_a = \left(\frac{v}{v - v_S}\right) \times n = \left(\frac{340}{340 - 20}\right) \times 1000 = 1063 \text{ Hz}$$

When source is receding, the frequency heard

$$n_r = \left(\frac{v}{v + v_S}\right) \times n = \frac{340}{340 + 20} \times 1000 = 944$$

$\Rightarrow \quad n_o : n_r = 9 : 8$

Short tricks: $\frac{n_a}{n_r} = \frac{v + v_S}{v - v_S} = \frac{340 + 20}{340 - 20} = \frac{9}{8}$

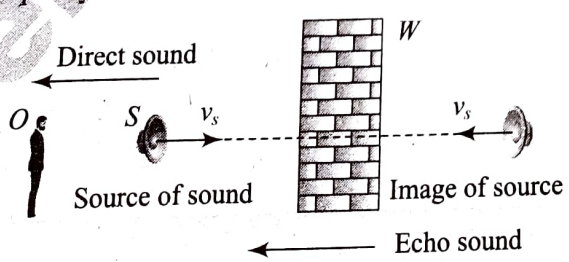
(Speed of sound v = 340 m/s)

4. (c) For source $v_S = r\omega = 0.70 \times 2\pi \times 5 = 22$ m/sec

Minimum frequency is heard when the source is receding the man. It is given by $n_{min} = n\frac{v}{v + v_s}$

$$= 1000 \times \frac{352}{352 + 22} = 941 \text{ Hz}$$

5. (b) For direct sound source is moving away from the observes, so frequency heard in this case



$$n_1 = n\left(\frac{v}{v + v_s}\right) = 500\left(\frac{332}{332 + 2}\right) = 500\left(\frac{332}{334}\right) \text{Hz}$$

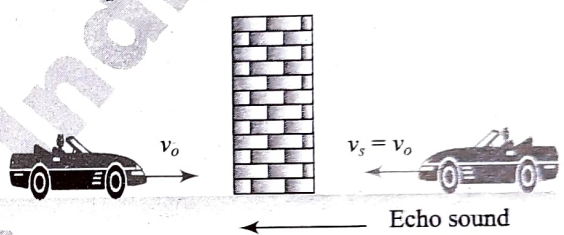
The other sound is echo, reaching the observer from the wall and can be regarded as coming from the image of source formed by reflection at the wall. This image is approaching the observer in the direction of sound.

Hence for reflected sound, frequency heard by the observer is

$$n_2 = n\left(\frac{v}{v - v_S}\right) = 500\left(\frac{332}{332 - 2}\right) = 500\left(\frac{332}{330}\right) \text{Hz}$$

Beats frequency $= n_2 - n_1 = 500 \times 332\left(\frac{1}{330} - \frac{1}{334}\right) = 6$

6. (c) Similar to previous question



The frequency of reflected sound heard by the driver

$$n' = n\left(\frac{v - (-v_0)}{v - v_s}\right) = n\left(\frac{v + v_0}{v - v_s}\right)$$

$$= 124\left[\frac{330 + (72 \times 5/18)}{330 - (72 \times 5/18)}\right] = 140 \text{ vibration/sec}$$

7. (c) When train is approaching frequency heard by the observer is

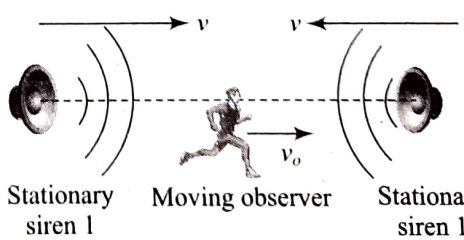
$$n_a = n\left(\frac{v}{v - v_S}\right) \Rightarrow 219 = n\left(\frac{340}{340 - v_S}\right) \quad \text{(i)}$$

When train is receding (goes away), frequency heard by the observer is

$$n_r = n\left(\frac{v}{v + v_s}\right) \Rightarrow 184 = n\left(\frac{340}{340 + v_s}\right) \quad \text{(ii)}$$

On solving equations (i) and (ii) we get n = 200 Hz and v_s = 29.5 m/s

8. (b) Observer is moving away from siren 1 and towards the siren 2.



Hearing frequency of sound emitted by siren 1

$$n_1 = n\left(\frac{v - v_0}{v}\right) = 330\left(\frac{330 - 2}{330}\right) = 328 \text{ Hz}$$

Hearing frequency of sound emitted by siren 2

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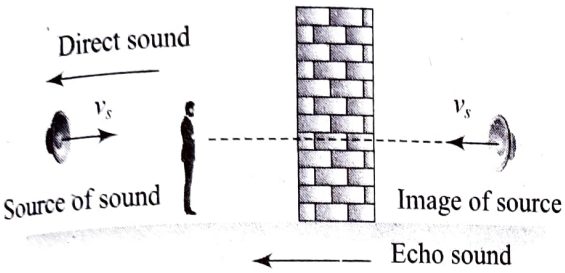
$$n_2 = n\left(\frac{v+v_0}{v}\right) = 330\left(\frac{330+2}{330}\right) = 332 \text{ Hz}$$

Hence, beat frequency $= n_2 - n_1 = 332 - 328 = 4$

9. (b) At point A, source is moving away from observer so apparent frequency $n_1 < n$ (actual frequency). At point B source is coming towards observer so apparent frequency $n_2 > n$ and point C source is moving perpendicular to observer so $n_3 = n$.

Hence $n_2 > n_3 > n_1$

10. (a) The observer will hear two sound, one directly from the source and other from reflected image of sound



Hence number of beats heard per second

$$= \left(\frac{v}{v-v_S}\right)n - \left(\frac{v}{v+v_S}\right)n$$

$$= \frac{2nvv_S}{v^2 - v_S^2} = \frac{2\times 256\times 330\times 5}{335\times 325} = 7.8 \text{ Hz}$$

11. (a) When a listener moves towards a stationary source, apparent frequency

$$n' = \left(\frac{v+v_0}{v}\right)n = 200 \quad \text{(i)}$$

When listener moves away from the same source

$$n'' = \frac{(v-v_O)}{v}n = 160 \quad \text{(ii)}$$

From (i) and (ii)

$$\frac{v+v_O}{v-v_O} = \frac{200}{160} \Rightarrow \frac{v+v_O}{v-v_O} = \frac{5}{4} \Rightarrow v = 360 \text{ m/sec}$$

Fill in the Blanks Type

12. $n' = n\left(\frac{v}{v-v_0}\right) = 450\left(\frac{340}{340-34}\right) = 500$ cycles/sec

13. $n' = n\left(\frac{v+v_O}{v}\right) \Rightarrow 2n = n\left(\frac{v+v_0}{v}\right) \Rightarrow \frac{v+v_0}{v} = 2$

$\Rightarrow \quad v_0 = v = 332$ m/sec

14. By Doppler's formula $n' = \frac{nv}{(v-v_S)}$

Since source is moving towards the listener so $n' > n$.

If $n = 100$, then $n' = 102.5$

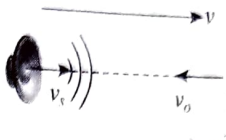
$\Rightarrow \quad 102.5 = \frac{100\times 320}{(320-v_S)} \Rightarrow v_s = 8$ m/sec

15. Since there is no relative motion between the observer and source, therefore there is no apparent change in frequency.

Hence, apparent frequency will be 200 Hz.

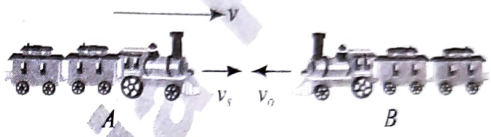
16. $n' = n\left(\frac{v}{v-v_S}\right) \Rightarrow \frac{n'}{n} = \frac{v}{v-v_S} \Rightarrow \frac{v}{v-v_S} = 3 \Rightarrow v_s = \frac{2v}{3}$

17. When source and listener both are moving towards each other then the frequency heard



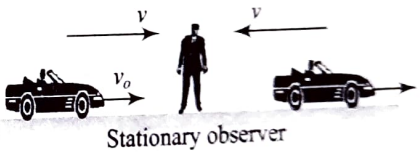
$$n' = n\left(\frac{v+v_O}{v-v_S}\right) \Rightarrow n' = f\left(\frac{v+v/10}{v-v/10}\right) = 1.22\, f$$

18. The frequency of whistle heard by passenger in the train B is



$$n' = n\left(\frac{v+v_0}{v-v_s}\right) = 600\left(\frac{340+15}{340-20}\right) \approx 666 \text{ Hz}$$

19. $n_{\text{Before}} = \frac{v}{v-v_c}n$ and $n_{\text{After}} = \frac{v}{v+v_c}\cdot n$



$$\frac{n_{\text{Before}}}{n_{\text{After}}} = \frac{11}{9} = \left(\frac{v+v_c}{v-v_c}\right) \Rightarrow v_c \Rightarrow \frac{v}{10}$$

20. Observer hears two frequencies

(i) n_1 which is coming from the source directly

(ii) n_2 which is coming from the reflection image of source

Hence, $n_1 = 680\left(\frac{340}{340-1}\right)$ and $n_2 = 680\left(\frac{340}{340+1}\right)$

$\Rightarrow n_1 - n_2 = 4$ beats

21. When engine approaches towards observer $n' = n\left(\frac{v}{v-v_s}\right)$

When engine going away from observer $n'' = \left(\frac{v}{v+v_S}\right)n$

$$\frac{n'}{n''} = \frac{v+v_s}{v-v_s} \Rightarrow \frac{5}{3} = \frac{340+v_s}{340-v_s} \Rightarrow v_s = 85 \text{ m/s}$$

DPP 8.2

Subjective Type

1. Let the velocities of car 1 and car 2 be V_1 m/s and V_2 m/s

$\therefore$ Apparent frequencies of sound emitted by car 1 and car 2 as detected at end point are

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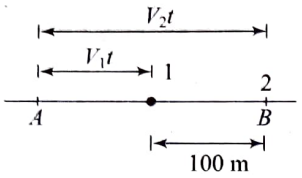
$$f_1 = f_0\frac{V}{V-V_1},\ f_2 = f_0\frac{V}{V-V_2}$$

$$\therefore\quad 330 = 300\frac{330}{330-V_1},\ 360 = 300\frac{330}{330-V_2}$$

$V_1 = 30$ m/s and V_2 100 m/s

The distance between both the cars just when the 2nd car reach and point B (as shown in the figure is)



100 m $= V_2t - V_1t \Rightarrow t = 4$ sec

2. λ_i = wavelength of the incident sound $= \dfrac{10u - \frac{u}{2}}{f} = \dfrac{19u}{2f}$

f_1 = frequency of the incident sound $= \dfrac{10u-u}{10u-\frac{u}{2}}f = \dfrac{18}{19}$

$= f_r$ = frequency of the reflected sound

λ_r = wavelength of the reflected sound

$$= \frac{10u+u}{f_r} = \frac{11u}{18f}\times 10 = \frac{11\times 19}{18}\cdot\frac{u}{f}$$

$$\frac{\lambda_i}{\lambda_r} = \frac{19u}{2f}\times\frac{18f}{11\times 19u}\times\frac{9}{11}$$

3. Frequency appearing before collision

$$= 300\left(\frac{330+10}{330-20}\right) = 1105$$

Frequency appearing after collision

$$= 300\left(\frac{330-20}{330-5}\right) = 961$$

So their ratio $= \dfrac{f_i}{f_a} = \dfrac{1105}{961}$

Single Correct Answer Type

4. (b) For observer approaching a stationary source

$$n' = \frac{v+v_0}{v}\cdot n \text{ and given } v_0 = at \Rightarrow n' = \left(\frac{an}{v}\right)t + n$$

This is the equation of straight line with positive intercept n and positive slope $\left(\dfrac{n}{v}\right)$.

5. (a) When the train is approaching the stationary observer frequency heard by the observer $n' = \dfrac{v+v_0}{v}n$

When the train is moving away from the observer, then frequency heard by the observer $n'' = \dfrac{v-v_0}{v}n$

It is clear that n' and n'' are constant and independent of time. Also and $n' > n''$.

6. (b) Wavelength remains the same during approach and recede. Hence (b).

7. (a) $f = f_0\left(1+\dfrac{V_{ob}}{V_{sound}}\right) \Rightarrow \dfrac{f}{f_0} = 1+\dfrac{V_{ob}}{V_{sound}}$ (straight line); when

$\dfrac{V_{ob}}{V_{sound}} = 0\ ;\ \dfrac{f}{f_0} = 1$

and as $\dfrac{V_{ob}}{V_{sound}} \to 1 \quad \Rightarrow \dfrac{f}{f_0} \to 2$

8. (b) When the source approaches the observer

Apparent frequency $n' = \dfrac{v}{v-v_s}\cdot n = n\left[\dfrac{1}{1-\frac{v_s}{v}}\right]$

$$= n\left[1-\frac{v_s}{v}\right]^{-1} = n\left[1+\frac{v_s}{v}\right]$$

(Neglecting higher powers because $v_S << v$)

When the source recedes the observed apparent frequency

$$n'' = n\left[1-\frac{v_s}{v}\right]$$

Given $n' - n'' = \dfrac{2}{100}n$, $v = 300$ m/sec

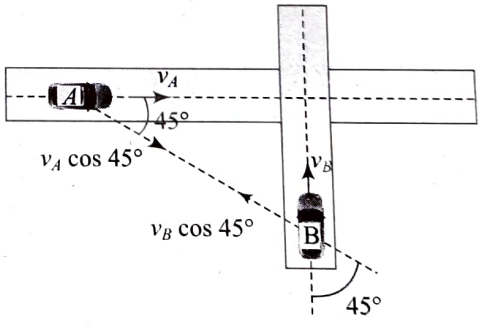
$$\therefore\quad \frac{2}{100}n = n\left[1+\frac{v_s}{v}\right] - n\left[1-\frac{v_s}{v}\right] = n\left[2\frac{v_s}{v}\right]$$

$$\Rightarrow\quad \frac{2}{100} = 2\frac{v_s}{v} \Rightarrow v_s = \frac{v}{100} = \frac{300}{100} = 3 \text{ m/sec}$$

9. (b) Here $v_A = 72$ km/hr $= 20$ m/sec

$v_B = 36$ km/hr $= 10$ m/sec

$$n' = n\left(\frac{v+v_B\cos 45°}{v-v_A\cos 45°}\right)$$



$$\Rightarrow\quad n' = 280\left(\frac{340+10/\sqrt{2}}{340-20/\sqrt{2}}\right) = 298 \text{ Hz}$$

10. (b) n_1 = Frequency of the police car horn observer heard by motorcyclist

n_2 = Frequency of the siren heard by motorcyclist

v_2 = Speed of motor cyclist

$$n_1 = \frac{330-v}{330-22}\times 176,\ n_2 = \frac{330+v}{330}\times 165$$

$$\because\quad n_1 - n_2 = 0 \Rightarrow v = 22 \text{ m/s}$$

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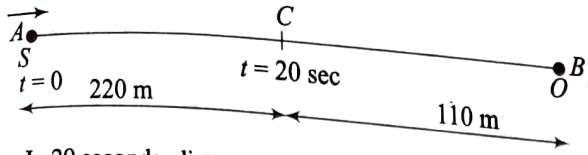
11. (d)



In 20 seconds, distance travelled by the source is:

$$S = \frac{1}{2}(1.110(20)^2 = 220 \text{ m}$$

Let $t = 0$ be the starting time.

The sound wave started at $t = 0$ from 'A' reaches the observer at 'B' after $\left(\frac{330}{330}\right) = 1$ sec, i.e., observer started hearing the sound at $t = 1$ sec. At $t = 20$ sec the source reaches at 'C' 220 m from 'A'.

The last sound wave starting from 'C' reaches 'B' after $\left(\frac{110}{330}\right) = \frac{1}{3}\text{sec} < 1\text{ sec}$.

Hence, the observer does not hear the sound for whole 20 sec, but for:

$$T = \left(19 + \frac{1}{3}\right)\text{sec} = \frac{58}{3}\text{sec}$$

Multiple Correct Answers Type

12. (b, c, d)

Frequency of B heard by A

$$f_A = 600\left(\frac{C + V_0}{C}\right) = 600\left(1 + \frac{2}{300}\right) = 604 \text{ Hz}$$

Beats heard by A = 8 beats/s

Frequency of A as heard by B

$$f_B = 596\left[\frac{C}{C - V_s}\right] = \frac{596}{298} \times 300 = 600 \text{ Hz}$$

Hence no beats are heard by B.

13. (a, b, c)

Number of waves striking the surface per second (or the frequency of the waves reaching surface of the moving target)

$$n' = \frac{(c + v)}{\lambda} = \frac{v(c + v)}{c}$$

Now these waves are reflected by the moving target (Which now acts as a source). Therefore apparent frequency of reflected second $n'' = \left(\frac{c}{c - v}\right)n' = v\left(\frac{c + v}{c - v}\right)$

The wavelength of reflected wave $= \frac{c}{n''} = \frac{c(c - v)}{v(c + v)}$

The number of beats heard by stationary listener

$$= n'' - v = v\left(\frac{c + v}{c - v}\right) - v = \frac{2vv}{(c - v)}$$

Hence option (a) is correct.

14. (b, c, d)

Frequency of horn directly heard by observer $\frac{v + v_0}{v + v_c} f$

Frequency of echo $= \frac{v}{v + v_c} f$

Frequency of echo of horn as heard by the observer.

$$\frac{v}{v - v_c} f \cdot \left(\frac{v + v_0}{v}\right)$$

Frequency of beats:

$$= (v + v_0) f \left\{\frac{1}{v - v_c} - \frac{1}{v + v_c}\right\} = \frac{2v_c(v + v_0)}{(v^2 - v_c^2)} f$$

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